Please write solutions to two of these problems. Solutions are due on **Thursday, September 17**. Solutions will be graded both for mathematical content and for exposition, which should be in complete English sentences. You need not show every computational detail, but please give enough information to allow readers to follow your reasoning. You may also include appropriate diagrams, which should be carefully drawn and labeled.

For this problem set, students *must* work in teams and each team will hand in one set of solutions. Each team should ideally have two students (let me know if this causes difficulty).

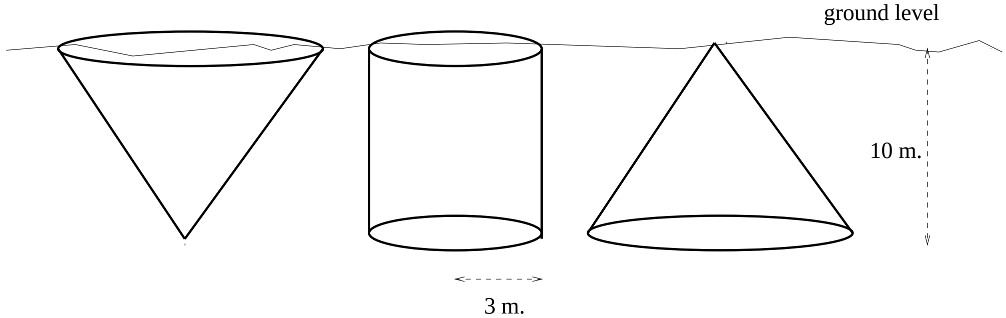
1. A freely falling body starting from rest has velocity $v = gt$ and displacement $s = \frac{1}{2}gt^2$ where t is the time elapsed since rest. Suppose the freely falling body starts at rest and falls 1,000 feet.

a) Calculate the time T (in seconds) this takes (here $g = 32$ ft/s^2) and the *time average* of the velocity of the body: $v_{\text{time aver}} = \frac{1}{T}\int_0^T v(t)\,dt$. Draw a graph of the function $v(t)$ for $0 \le t \le T$. Find the time t when $v(t) = v_{\text{time aver}}$ and give a graphical interpretation.

b) Find a formula for the velocity as a function $f(s)$ of displacement s, and calculate the *distance average* of the velocity: $v_{\text{dist aver}} = \frac{1}{1000}\int_0^{1000} f(s)\,ds$. Draw a graph of the function $v = f(s)$ for $0 \le s \le 1000$. Find the distance s that the body has fallen when $f(s) = v_{\text{dist aver}}$ and give a graphical interpretation.

Note $v_{\text{dist aver}} \ne v_{\text{time aver}}$! Every user of statistics (this means, essentially, every person in this course) should do this problem. Averages can be difficult to understand.

2. A homogeneous liquid whose density is 300 kg/m^3 fills three buried containers. The containers, drawn below, are each 10 meters tall. The top of each container is at ground level. All three containers have the *same* volume. The middle container is a cylinder, and the other two are circular cones. Which container needs the *least* amount of work to empty (that is, to pump the liquid to ground level)? Which container needs the *most* work to empty? Justify your assertions by computing the work necessary in each case. You may also discuss *why* your answer is correct!



Note You probably want to begin with the cylinder first.

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3. The following statements are facts:

$$(10{,}000{,}000{,}000)^{\left(\frac{1}{10{,}000{,}000{,}000}\right)} \approx 1.\ 00000\ 000\underline{23}\ \underline{02585}\ \underline{09}564$$

and

$$\ln 10 \approx \underline{2}.\underline{30258}\ \underline{509}29$$

Explain the amazing coincidence of the digits.

Hint What is A^B in a calculus course? Then approximate e^x when x is small.

4. A torus (doughnut) is created by revolving the area inside a circle of radius r around a central axis which is a distance R from the center of the circular region. Find a formula (which will mention both r and R) for the resulting volume.

Below are two diagrams. On the left is an oblique view of the torus and an oblique view of one of the circular regions. The right diagram is a cross-section of the torus, perpendicular to the central axis and the center of the circular region).