Some (more) pictures for 152H

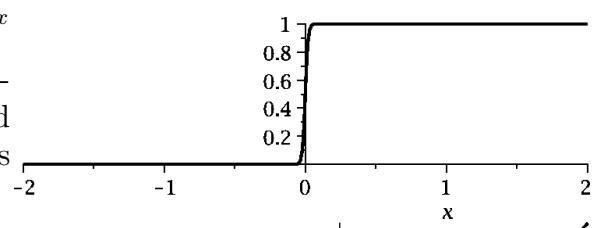
11/2/2009

The fifth problem on the fifth workshop was this:

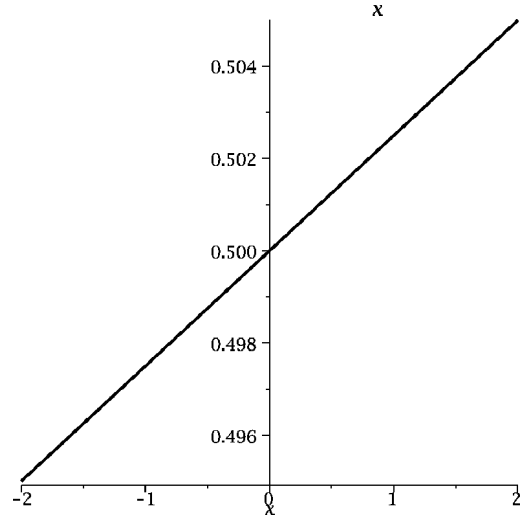
> Consider the function $F(x) = \dfrac{e^{Ax}}{1+e^{Ax}}$ where A is a constant.
>
> a) Sketch $y = F(x)$ for x in $[-1, 1]$ if $A = 10^{10}$. Explain why your drawing is correct.
>
> b) Sketch $y = F(x)$ for x in $[-1, 1]$ if $A = 10^{-10}$. Explain why your drawing is correct.
>
> c) Sketch $y = F(x)$ for x in $[-1, 1]$ if $A = -10^{10}$. Explain why your drawing is correct.
>
> d) Sketch $y = F(x)$ for x in $[-1, 1]$ if $A = -10^{-10}$. Explain why your drawing is correct.
>
> **This is today's "biology" problem. A graphing device probably can't help much with this – you'll need "pure thought".**

Only one student chose to do this problem, and I didn't want to let it slide by without more notice. Chemical and biochemical reactions sometimes have "rate constants" which are very large or very small, and the resulting curves may seem rather strange initially. I thought I would show you some sample graphs in the interval $[-2, 2]$, when the constants involved are not so exaggerated but are large enough to show what's going on.

Here's a graph of $\dfrac{e^{100x}}{1+e^{100x}}$. If x is positive, e^{100x} is likely to be enormous (at $x = .2$, e^{100x} is almost $5 \cdot 10^8$) and $\frac{\texttt{large}}{1+\texttt{large}}$ should be about $\frac{1}{2}$. And when x is negative, the fraction is near 0. Is this picture explained well enough?



Now consider $\dfrac{e^{\frac{1}{100}x}}{1+e^{\frac{1}{100}x}}$. Look at the scales on the axes *carefully*. For x in $[-2, 2]$, $e^{\frac{1}{100}x}$ is $e^{\frac{\texttt{SMALL}}{100}}$. This is (using the tangent line approximation $e^x \approx 1 + x$ valid for x near) $1 + \frac{\texttt{SMALL}}{100}$ so that the function is $\frac{1+\frac{\texttt{SMALL}}{100}}{1+(1+\frac{\texttt{SMALL}}{100})} \approx \frac{1}{2} + \frac{1}{400}\texttt{SMALL}$. The last approximation again uses the tangent line: $\frac{1+\frac{x}{100}}{2+\frac{x}{100}} \approx \frac{1}{2} + \frac{1}{400}x$ for x near 0. With a more standard vertical scale, the graph "is" nearly a horizontal line through $\left(0, \frac{1}{2}\right)$ in $[-2, 2]$.



Graphs for -100 and $-\frac{1}{100}$ are left-to-right reflections of these.