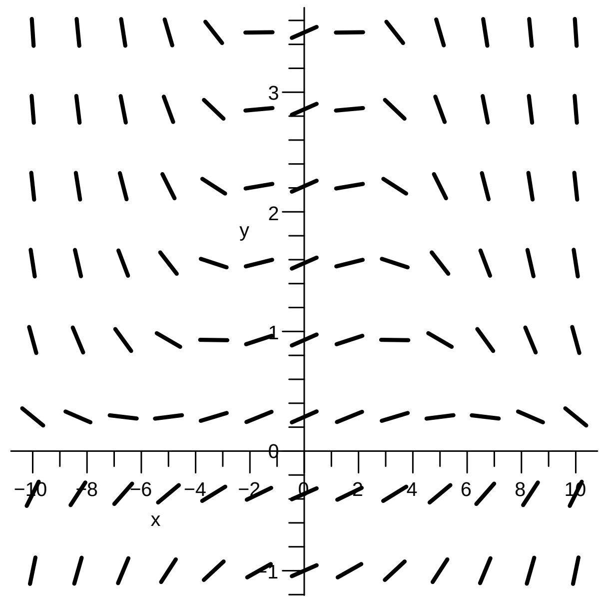
1. The horizontal and vertical axes on this graph have different scales. x goes from -10 to 10 and y goes from -1 to 3.5. The graph is a direction field for the differential equation $y' = \frac{1}{10}\left(1 - \frac{1}{10}yx^2\right)$.



a) Sketch the solution curve passing through $(0, 1)$ **on the graph**.

b) How many critical points does this solution curve seem to have? What types of critical points do they seem to be? If (x_0, y_0) is a critical point, find an exact algebraic relationship between x_0 and y_0.

Comment The equation *can't* be solved in terms of standard functions. Use information from the graph and the differential equation.

2. Let $N(t)$ be the fraction of the population who have heard a given piece of news t hours after its initial release. According to one model, the rate $N'(t)$ at which the news spreads is equal to k times the fraction of the population that has not yet heard the news, for some constant k.

a) Determine the differential equation satisfied by $N(t)$.

b) Find the solution of this differential equation with the initial condition $N(0) = 0$ in terms of k.

c) Suppose that half of the population is aware of an earthquake 8 hours after it occurs. Use the model to calculate k and estimate the percentage that will know about the earthquake 12 hours after it occurs. (This is problem 22 in section 9.2 of the textbook.)

3. **Air Resistance** A projectile of mass $m = 1$ travels straight up from ground level with initial velocity v_0. Suppose that the velocity v satisfies $v' = -g - kv$.

a) Find a formula for $v(t)$.

b) Show that the projectile's height $h(t)$ is given by $h(t) = C(1 - e^{-kt}) - \dfrac{g}{k}t$ where $C = k^{-2}(g + kv_0)$.

c) Show that the projectile reaches its maximum height at tune $t_{\max} = k^{-1}\ln(1 + kv_0/g)$.

d) In the absence of air resistance, the maximum height is reached at time $t = v_0/g$. In view of this, explain why we should expect that $\displaystyle\lim_{k\to 0} \frac{\ln(1 + \frac{kv_0}{g})}{k} = \frac{v_0}{g}$.

e) Verify the previous equation. (This is problem 26 in section 9.2 of the textbook. Look there for a *Hint* about how to verify the equation.)