1. A freely falling body starting from rest has velocity $v = gt$ and displacement $s = \frac{1}{2}gt^2$ where t is the time elapsed since rest. Suppose the freely falling body starts at rest and falls 1,000 feet. Here $g = 32$ ft/s^2.

a) Calculate the time T (in seconds) this takes and the *time average* of the velocity of the body: $v_{\text{time aver}} = \frac{1}{T}\int_0^T v(t)\,dt$. Draw a graph of the function $v(t)$ for $0 \le t \le T$. Find the time t when $v(t) = v_{\text{time aver}}$ and give a graphical interpretation.

b) Find a formula for the velocity as a function $f(s)$ of displacement s, and calculate the *distance average* of the velocity: $v_{\text{dist aver}} = \frac{1}{1000}\int_0^{1000} f(s)\,ds$. Draw a graph of the function $v = f(s)$ for $0 \le s \le 1000$. Find the distance s that the body has fallen when $f(s) = v_{\text{dist aver}}$ and give a graphical interpretation.

Note $v_{\text{dist aver}} \ne v_{\text{time aver}}$! Averages can be difficult to understand.

2. A sort of raindrop is obtained by revolving the profile curve

$$y = \sqrt{x}(x - C)^2 \text{ for } 0 \le x \le C$$

about the x-axis. Here C is a positive constant.

a) Sketch the profile curve and the solid of revolution.

b) For which value of C will the raindrop have volume 1? What are the approximate dimensions (length and diameter) of this raindrop?

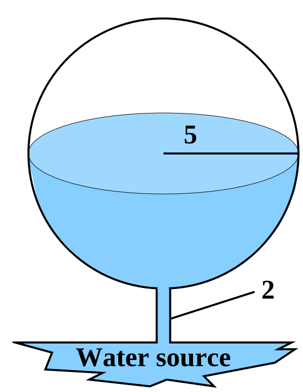
3. Electrons repel each other with a force which is inversely proportional to the square of the distance between them; call the proportionality constant k in the units to be used. Suppose one electron is fixed at $x = 0$ on the $x-$axis.

a) Find the work done in moving a second electron along the $x-$axis from the point $x = 10$ to the point $x = 1$.

b) Find the work done in moving the second electron along the $x-$axis from the point $x = M$ to the point $x = 1$.

c) What happens to your answer in b) (which should depend on M) as $M \to +\infty$?

4. Water is pumped into a spherical tank of radius 5 ft from a source located 2 ft below a hole at the bottom (figure to the right). The density of water is 64.2 lb/ft^3.



a) Calculate the work required to fill the tank.

b) Calculate the work $F(h)$ required to fill the tank to a height h ft from the bottom of the sphere.

(This is quoted from problems #41 and #42 of the Chapter Review Exercises in Chapter 6 of the text.)