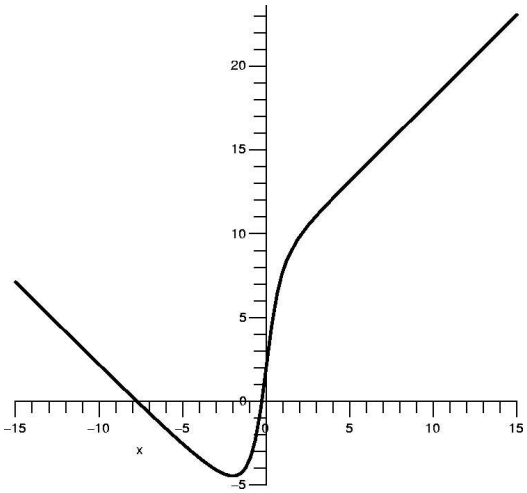
(12) 1. In this problem, $f(x) = \frac{1}{4}x^4 - \frac{1}{3}x^3 - x^2$.

Find the absolute minimum and absolute maximum values of f in the interval $[-2, 1]$.

(8) 2. Suppose $g(x)$ is a differentiable function, and we know that $g(1) = 2$, $g'(1) = 3$, and $g''(1) = -1$. Define the function $h(x)$ by the equation $h(x) = g(2x^3 - x^2)$. Compute $h(1)$, $h'(1)$, and $h''(1)$.

(12) 3. Suppose the sum of two non-negative numbers is 15. Verify using calculus that the largest the product of the square of one multiplied by the other can be is 500.

(20) 4. Here $f(x) = \dfrac{x^2 + 8x + 2}{\sqrt{x^2 + 1}}$. A `Maple` graph of $y = f(x)$ is shown to the right. These equations are true: $f'(x) = \dfrac{x^3 + 8}{(x^2 + 1)^{3/2}}$ and $f''(x) = \dfrac{3x(x - 8)}{(x^2 + 1)^{5/2}}$.



a) Compute $\lim_{x \to +\infty} f(x)$ and $\lim_{x \to -\infty} f(x)$. Give evidence (not just the graph!) supporting your assertions.

b) Where is f increasing? Where is f decreasing? Support your assertions using results from the course and information supplied (not just the graph!).

c) Exactly how many roots does the equation $f(x) = 100$ have? Do *not* try to compute these roots but use results from the course and your answers to a) and b) to verify your answer.

d) Where is the graph of $y = f(x)$ concave up and concave down? Label any points of inflection <u>on the graph</u>. (You need not give support for your answer here but indicate the intervals clearly below, and label the graph as needed.)

(20) 5. Suppose $f'(x) = x(x-1)^2(x-2)^3(x-3)$.

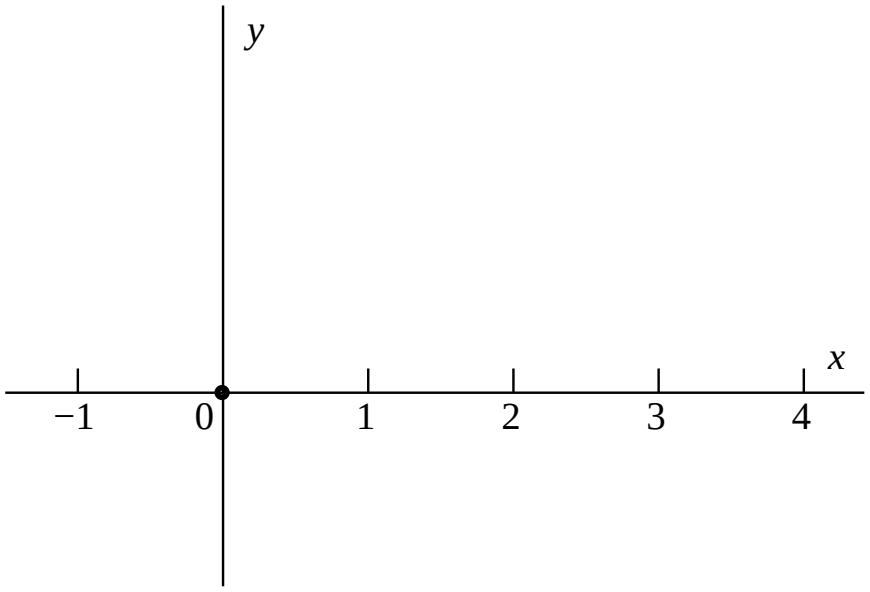
Comment and advice This is a formula for the derivative of f. Please do *not* "expand" or "simplify" the formula, and do *not* try to find f explicitly.

a) Where are the critical points of f and what are their types? Enter the information in the table below. The types should be selected from the words MAXIMUM, MINIMUM, and NEITHER. *Briefly* give reasoning which supports your assertions about this function in the space after the table.

Critical point: $x =$				
Type of critical point				

Explanation

b) Suppose you know that $f(0) = 0$. Sketch a qualitatively correct graph of $y = f(x)$ on the interval $[-1, 4]$ using the axes below. To help, a point on the graph at $(0, 0)$ has already been drawn.



(12) 6. Find the equations of all vertical and horizontal asymptotes of

$$f(x) = \frac{3e^x + 5}{7e^x - 2}.$$

Numerical approximations are *not* acceptable.

(8) 7. A car moves along a road from the town of FROG to the town of TOAD. Its speed is always between 40 and 60 mph. The car leaves FROG at 9 AM and enters TOAD at 10:30 AM.

Write a mathematical model of this situation (include information about units in your model!). Your model should include a function with a domain you define, with properties of the function stated carefully. Use this model together with *specific results* from this course to obtain some estimate of the distance along the road from FROG to TOAD.

(8) 8. Suppose that $F(x) = (2x^3 + 8)^{12} - (7 - 3x^5)^{15}$.

a) Compute $F'(x)$. Explain why if x is positive, then $F'(x)$ must also be positive.

b) Use your answer to a) and calculus to explain briefly why $F(200)$ is less than $F(400)$.

Second Exam for Summer Bridge: Foundations of Calculus

July 26, 2013

NAME _____

Do all problems, in any order.
Show your work. An answer alone may not receive full credit.
No notes other than the distributed formula sheet may be used on this exam.
No calculators or other electronic devices may be used on this exam.

Problem Number	Possible Points	Points Earned:
1	12	
2	8	
3	12	
4	20	
5	20	
6	12	
7	8	
8	8	
Total Points Earned:		