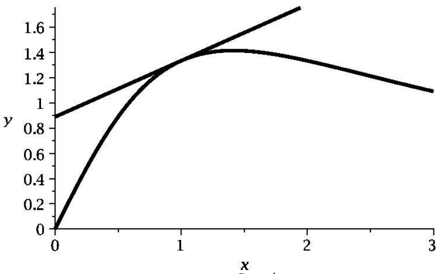
(16) 1. In this problem, $f(x) = \frac{4x}{2+x^2}$. Below is a graph of $y = f(x)$ on the interval $[0, 3]$.



a) Find an equation for the line tangent to $y = f(x)$ when $x = 1$. Sketch that line as well as you can on the graph displayed to the right. **Answer** Here $f'(x) = \frac{4(2+x^2)-2x(4x)}{(2+x^2)^2} = \frac{8-4x^2}{(2+x^2)^2}$ so that $f'(1) = \frac{4}{9}$. Also $f(1) = \frac{4}{3}$. An equation for the line is therefore $y - \frac{4}{3} = \frac{4}{9}(x-1)$ or, if you wish, $y = \frac{4}{9}x + \frac{8}{9}$. The tangent line is shown on the graph.

b) What are the coordinates of the local maximum (highest point) shown on the graph? (You need *not* simplify your answer but you do need to give both coordinates!) **Answer** The bump occurs where $f'(x) = 0$. The top of $f'(x)$ is $8 - 4x^2$ and is 0 at $\pm\sqrt{2}$. We use $\sqrt{2}$, and $f(\sqrt{2}) = \frac{4\sqrt{2}}{2+(\sqrt{2})^2} = \sqrt{2}$. The point is $(\sqrt{2}, \sqrt{2})$.

(10) 2. Suppose $f(x) = \frac{1}{3x+2}$. Use the **definition of derivative** to find $f'(x)$.

Answer $f(x+h) - f(x) = \frac{1}{3(x+h)+2} - \frac{1}{3x+2} = \frac{(3x+2)-(3(x+h)+2)}{(3(x+h)+2)\cdot(3x+2)} = \frac{3x+2-3x-3h-2}{(3(x+h)+2)\cdot(3x+2)} = -\frac{3h}{(3(x+h)+2)\cdot(3x+2)}$. In $\frac{f(x+h)-f(x)}{h}$ the h's drop out and we get $-\frac{3}{(3(x+h)+2)\cdot(3x+2)}$. The limit as $h \to 0$ is $-\frac{3}{(3x+2)^2}$. (The Quotient Rule can check this!)

(12) 3. a) Find the derivative of $3 - 12x^7 + 5\sin x$. (Full credit for the answer alone – please don't simplify.) **Answer** $0 - 12(7x^6) + 5\cos x$

b) Find the derivative of $(7x + 16\cos x)(x^3 + 17)$. (Full credit for the answer alone – please don't simplify.) **Answer** $(7 - 16\sin x)(x^3 + 17) + (7x + 16\cos x)(3x^2 + 0)$

c) Find $F'(2)$ if $F(x) = x^3 f(x)$ and f is differentiable with $f(2) = -3$ and $f'(2) = -1$. (Show your work in this part.) **Answer** $F'(x) = 3x^2 f(x) + x^3 f'(x)$ (Product Rule) so that $F'(2) = 3(2^2)(-3) + 2^3(-1) = -44$.

(16) 4. Evaluate the indicated limits exactly. Give evidence to support your answers.

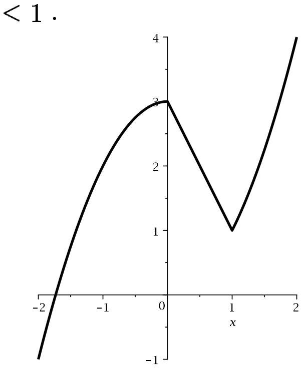
a) $\lim_{x\to 1} \frac{x^2+2x-3}{x-1}$ **Answer** $\frac{x^2+2x-3}{x-1} = \frac{(x+3)(x-1)}{x-1} = x + 3$ if $x \neq 1$. So the limit is $\lim_{x\to 1} x + 3 = 4$.

b) $\lim_{x\to 2^+} \frac{|x-1|-1}{|x-2|}$ **Answer** For $x \to 2^+$, $x > 2$. Then $x - 1$ and $x - 2$ are both positive, so that $|x-1| = x - 1$ and $|x-2| = x - 2$. Then $\frac{|x-1|-1}{|x-2|} = \frac{x-1-1}{x-2} = 1$ and the limit is 1.

c) $\lim_{x\to 3} \frac{\sqrt{x}-\sqrt{3}}{x-3}$ **Answer** For $x \neq 3$, $\frac{\sqrt{x}-\sqrt{3}}{x-3} = \frac{\sqrt{x}-\sqrt{3}}{(\sqrt{x}-\sqrt{3})(\sqrt{x}+\sqrt{3})} = \frac{1}{\sqrt{x}+\sqrt{3}}$. Then the limit as $x \to \sqrt{3}$ just becomes $\frac{1}{2\sqrt{3}}$.

d) $\lim_{x\to 4} \frac{3x-2}{\cos(\pi x)}$ **Answer** Since $\cos(4\pi) = 1$ and is *not* 0, the function involved is continuous at $x = 4$ and the limit's value is 10 (just "plug in").

(12) 5. Suppose that the function $f(x)$ is described by $f(x) = \begin{cases} 3 - x^2 & \text{if } x < 0 \\ Ax + B & \text{if } 0 \leq x < 1 \\ x^2 & \text{if } 1 < x \end{cases}$.



a) Find A and B so $f(x)$ is continuous for all numbers. Show work supporting your answer. **Answer** For any A and B, the function is continuous when $x \neq 0$ and $x \neq 1$. We check what happens at 0 and 1. $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} 3 - x^2 = 3$ and $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} Ax + B = B$. Select $B = 3$ to insure continuity at 0. Also $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} Ax + B = A + B = A + 3$ and $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} x^2 = 1$ so A must be -2.

b) Sketch $y = f(x)$ on the axes given for the values of A and B found in a) when x is in the interval $[-2, 2]$. **Answer** Done and displayed to the right.

(8) 6. Suppose that $f(x) = x^5 + 3\cos(Kx^2)$ where K is an unknown constant.

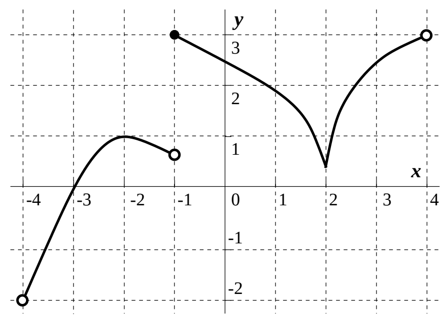
a) Briefly explain why $f(2)$ must be positive. **Answer** The values of cosine are between -1 and 1. Therefore $f(2) = 2^5 + 3\cos(4K) \geq 2^5 - 4 = 28 > 0$.

b) Briefly explain why $f(-2)$ must be negative. **Answer** Similarly, since the values of cosine are between -1 and 1, $f(-2) = (-2)^5 + 3\cos(-4K) \leq (-2)^5 + 4 = -28 < 0$.

c) Briefly explain why the equation $f(x) = 0$ must have a solution, and specify an interval on the x-axis in which a solution can be found. **Answer** Since f is continuous, and f's values at -2 and 2 have opposite signs, the **Intermediate Value Theorem** implies that $f(x) = 0$ must have a solution on the interval $[-2, 2]$.

(8) 7. What is the domain of $f(x) = \frac{\sqrt{x+1}+\sqrt{4-x}}{\sin x}$? Explain your answer algebraically.
Answer $\sqrt{x+1}$ is defined if $x + 1 \geq 0$ which is $x \geq -1$. $\sqrt{4-x}$ is defined if $4 - x \geq 0$ which is $4 \geq x$. Therefore x certainly can only be in the interval $[-1, 4]$. But $\sin x$ should not be 0. The sine function is 0 at all multiples of $\pi \approx 3.14$. The only multiples of π in the interval $[-1, 4]$ are 0 ($\mathbf{0}\pi$!) and $\mathbf{1}\pi$. Those numbers must be excluded. So the domain of f is $[-1, 0)$ and $(0, \pi)$ and $(\pi, 4]$.

(18) 8. In this problem the function $f(x)$ has domain all x's between -4 and 4: $-4 < x < 4$. A graph of $y = f(x)$ is displayed [below] to the right. Answer the following questions as well as you can based on the information in the graph.



a) What is the range (the collection of values) of $f(x)$? **Answer** $(-2, 3]$. Why: what horizontal lines ($y = K$) intersect the graph of f?

b) For which x is $f(x)$ *not* continuous? **Answer** $x = -1$ only (the graph has a jump there).

c) For which x is $f(x) = 0$? **Answer** $x = -3$

d) For which x is $f(x) > 0$? **Answer** The interval $(-3, 4)$ (where the graph is above the horizontal axis).

e) For which x is $f(x)$ *not* differentiable? **Answer** $x = -1$ and $x = 2$. At -1, there's a jump, so f can't be differentiable since it is not continuous. At 2, the graph has a corner (a "cusp") so f is also not differentiable.

f) For which x is $f'(x) = 0$? **Answer** At $x = -2$ (the tangent line is horizontal).

g) For which x is $f'(x) > 0$? **Answer** The intervals $(-4, -2)$ and $(2, 4)$. The tangent lines slant "upward" in those intervals.

Here are statistics about the exam results:

Problem #	1	2	3	4	5	6	7	8	Total
Maximum	16	10	12	16	12	8	8	16	92
Minimum	3	9	8	7	0	1	1	7	52
Median	10	10	12	15	11	5	6	12	83
Mean	11.22	9.78	11.44	13.22	9.89	5.00	5.00	11.78	77.33

Letter grade "bins" for numerical results follow. However, students should realize that the numerical results will be what's used in computing the course grade, and the letter grade is only a rough guide.

A: [88,100]; B+: [82,87]; B: [73,81]; C+: [67,72]; C: [60,66]; D: [51,59]; F: [0,50].