The objective of this problem is to find a straight line $y = mx + b$ that best approximates the curve $y = e^x$ over the interval $0 \leq x \leq 1$. In order to find such a line, we must select a way of measuring the error associated with the approximation. Two possibilities, the functions $\mathbf{LIN}(m,b)$ and $\mathbf{QUAD}(m,b)$, have been suggested:

$$\mathbf{LIN}(m,b) = \int_0^1 \mathbf{ERR}(x)dx$$
$$\mathbf{QUAD}(m,b) = \int_0^1 \big(\mathbf{ERR}(x)\big)^2 dx$$

where

$$\mathbf{ERR}(x) = e^x - (mx+b)$$

is the difference between $e^x$ and $mx + b$.

If the curve and straight line cross in the interval $[0,1]$, $\mathbf{ERR}(x)$ will change sign in that interval. By plotting the curve and a line that seems to be a good approximation, we can see that $\mathbf{LIN}(x)$ is not a good measure of total error. The parameters $m$ and $b$ could be chosen so that the integral of the error obtained over the range of $x$ values for which $\mathbf{ERR}(x) > 0$ exactly cancels the integral over the range of $x$ values for which $\mathbf{ERR}(x) < 0$. If we tried to minimize $\mathbf{LIN}(m,b)$ as described in section 12.7 of the text, we'd find that the derivatives with respect to $m$ and $b$ were never zero, so we could not use this method to minimize **LIN**. In fact, **LIN** has *no* minimum.

$\mathbf{QUAD}(m,b)$ is a better measure of error since such cancellation can't occur because the integrand is always greater than or equal to 0. The method of section 12.7 can be used successfully here to find values of $m$ and $b$ that minimize $\mathbf{QUAD}(m,b)$.

1) Take derivatives of $\mathbf{QUAD}(m,b)$ with respect to $m$ and $b$.
2) Solve for values of $m$ and $b$ that make both derivatives zero (simultaneously).
3) Apply the second derivative test (Stewart, p.794) to verify that the values of $m$ and $b$ obtained correspond to a minimum of $\mathbf{QUAD}(m,b)$.

I used `Maple` to do these computations on an X terminal. Below is the record of a `Maple` session, together with the graph of $e^x$ and the resulting linear approximation.