Review ADVICE for the final exam in Math 135 (including some review problems)

After this introductory paragraph there are some problems covering the last 3 or 4 weeks of the course. The final is cumulative and may cover all the material in the course. There will be more emphasis, though, on the material of the last 3 or 4 weeks which has not yet been tested. So please look at the problems below. You should also review the following material, keeping in mind that if we thought it was important earlier, we probably still think it is important.

- The earlier review sheets (with some answers available on the web through the address below).
- The exams that you had earlier in this course.
- Two "real" exams given to one lecture class in this course (with answers and grading guide available on the web through the address below).

There's lots of Math 135 material which can be accessed through:

`http://math.rutgers.edu/~greenfie/currentcourses/math135/math135_index.html`

The time, date, and location of final exams in this course will be announced by the instructors in each class.

The problems below are intended to be representative of the type of problems which may appear on exams. Please realize that many routine review problems are available in the review sections of the text.

Graphing calculators may be used on the Math 135 final exam but calculators and computers with QWERTY keyboards or symbolic differentiation and integration programs are not allowed. Students may bring one sheet of paper containing notes (using both sides!) up to 8.5 by 11 inches in size to any exam.

1. Suppose $M(x) = x^5 + 7x + 6$.

a) Compute $\lim_{x\to+\infty} M(x)$ and $\lim_{x\to-\infty} M(x)$. Explain briefly why the results allow you to conclude that $M(x) = 0$ has at least one root.

b) Explain how to apply Newton's method to find a root of $M(x) = 0$: that is, describe how Newton's method gets a new approximation, x_{new}, to a root from an old one, x_{old}, using a formula of the form $x_{\text{new}} = G(x_{\text{old}})$. Describe G precisely for this M.

c) Start with an initial guess of 1 and repeat Newton's three times to try to get close to the root of $M(x) = 0$ (which has been reported by rumor to be approximately $-.8079567434$).

2. Suppose Q and its first derivative, Q', have the values indicated in the accompanying table. Use this table to find $Q(1)$ and $Q(Q(1))$ and $Q(Q(Q(1)))$ exactly. Use linear approximation (also known as differentials) and the information in this table to find approximate values of $Q(1.03)$, $Q(.98)$, $Q(Q(1.03))$, $Q(Q(.98))$, $Q(Q(Q(1.03)))$, and $Q(Q(Q(.98)))$. **Note** You could define $R(x) = Q(Q(x))$ and $S(x) = Q(Q(Q(x)))$ and then work with the functions Q, R, and S.

x	$Q(x)$	$Q'(x)$
0	1	-1
1	2	-2
2	0	5
3	1	2

3. Evaluate these indefinite integrals.

a) $\displaystyle\int 7x^2 - 3e^x + \frac{5}{x}\,dx$

b) $\displaystyle\int (x^3+5)^2\,dx$

c) $\displaystyle\int 5\sin x + \cos(5x)\,dx$

4. Find a solution to the differential equation $y' = 2x^3 - 1$ passing through the point $(2, 3)$.

5. What's the highest the curve $y = K(x)$ can be if $K''(x)$ is -9 always and if also $K(0) = 0$ and $K(2) = 0$?

6. Evaluate these definite integrals using methods of calculus.

a) $\displaystyle\int_1^2 \left(x^3 - \frac{1}{x^4}\right) dx$

b) $\displaystyle\int_0^{\ln 3} 4e^{2x}\, dx$

c) $\displaystyle\int_0^2 x^2\sqrt{1+3x^3}\, dx$

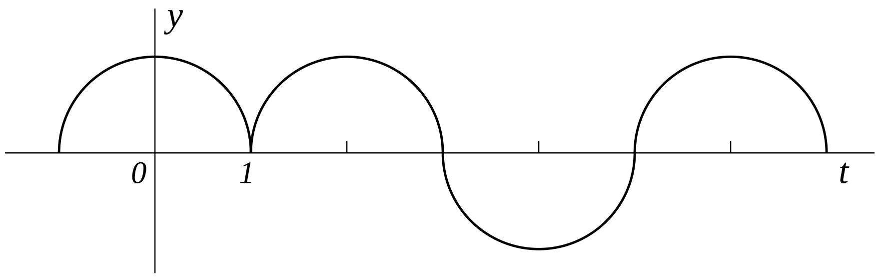
d) $\displaystyle\int_{\ln \pi}^{\ln 2\pi} e^x \sin(e^x)\, dx$

7. a) Compute the derivative of $e^x \sin x - e^x \cos x$.

b) Use your answer to a) to compute $\displaystyle\int_0^{\pi} e^x \sin x\, dx$.

8. If $F(x) = \displaystyle\int_{-42}^{x} \frac{\sin(t^2)}{1+t^4}\, dt$, compute $F(-42)$, $F'(0)$, and $F'(\sqrt{\pi})$ exactly.

9. Below is a graph of $y = f(t)$.



Graph of $y = f(t)$

Note that each segment of the graph is a semicircle. Suppose that $g(x) = \displaystyle\int_{-1}^{x} f(t)\, dt$.

a) What are $g(-1)$ and $g(0)$ and $g(5)$? What are $g'(-1)$ and $g'(0)$ and $g'(5)$?

b) Compute a Riemann sum estimate for $g(3)$ associated with the partition $-1 < 0 < 1 < 3$ where the sample points are the left endpoints of the subintervals.

10. What is the area enclosed by $y = x^5$, the x-axis, and the lines $x = 1$ and $x = 2$?

11. Find the area under the arch of $y = x(x-1)^2$. Stated more properly: what is the area of the bounded region in the plane bounded by the x-axis and $y = x(x-1)^2$?

On the web at `http://math.rutgers.edu/~greenfie/currentcourses/math135/math135_index.html`.