1 A

(8) 1. Evaluate the indicated limits exactly. Give brief evidence supporting your answers which is not based on a calculator graph or calculator computations.

a) $\displaystyle\lim_{x\to 3} \frac{2x^2-6x}{x^2-2x-3}$.

b) $\displaystyle\lim_{x\to +\infty} \frac{2x^2-6x}{x^2-2x-3}$.

(12) 2. Find the equations of all vertical and horizontal asymptotes of the function

$$f(x) = \frac{3}{(1+e^{-2x})(1-4e^{-5x})} .$$

Computations with exp and log should be simplified as much as possible. A numerical approximation like 1.40135 is **not** acceptable.

(12) 3. Write the definition of derivative as a limit and *use this definition* to find the derivative of $f(x) = 2x^2 + \dfrac{5}{x}$.

(14) 4. In this problem

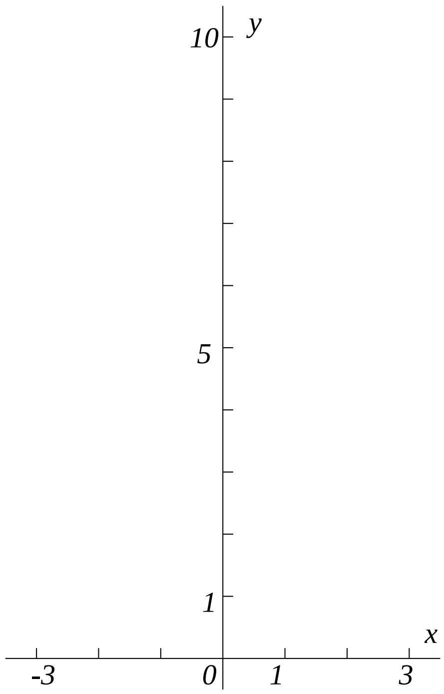
$$g(x) = \begin{cases} 1+x^2 & \text{if } x < -2 \\ A+Bx & \text{if } -2 \le x < 1 \\ x^2 & \text{if } x \ge 1 \end{cases}$$

where A and B are constants to be determined in part a).

a) Find A and B so that g is continuous at all points.

b) Sketch a graph of $y = g(x)$ on the axes displayed.

c) For which values of x is g *not* differentiable?
Explain your answer briefly.



(12) 5. Find the first derivative of each of the following functions.

a) $\dfrac{3x^3-1}{3x^4+1}$

b) $x^5 \sin x$

c) $e^{-\sqrt{x^2-1}}$

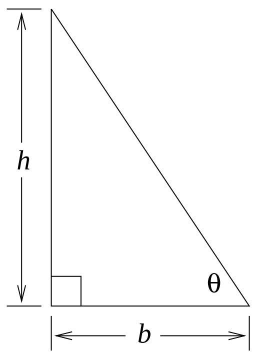
(10) 6. A function y is defined implicitly by the equation

$$x^2 - y^2 - 5xy + x + y = -10 .$$

a) Find y' in terms of x and y.

b) Find an equation for the tangent line to the graph of this function at the point $(1, 2)$.

(20) 7. Suppose that the right triangle with height h and base b (as shown here) *always* has an area of 6 square inches. At a certain time, the length of b is 3 inches and it is increasing at the rate of .04 inches per minute.



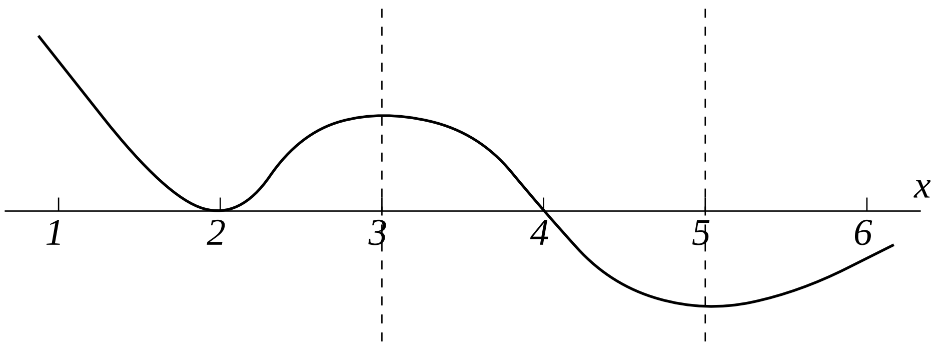
a) What is the length of h at that time?

b) How fast is the length of h changing at that time? Is it increasing or decreasing?

c) What is the angle θ at that time?

d) How fast is the angle θ changing at that time? Is it increasing or decreasing?

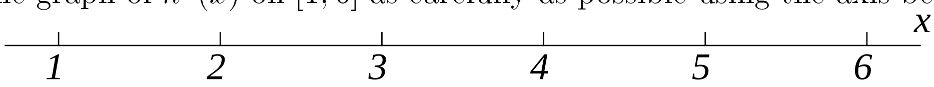
(26) 8. A certain function h is known to have continuous first and second derivatives on the interval $[1, 6]$. The graph of the first derivative, h', is given by:



The graph of $y = h'(x)$

a) Based on the evidence given, what are the intervals on which h is increasing? Based on the evidence given, what are the intervals on which h is decreasing? Briefly explain your assertions.

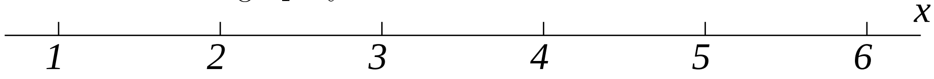
b) Sketch the graph of $h''(x)$ on $[1, 6]$ as carefully as possible using the axis below.



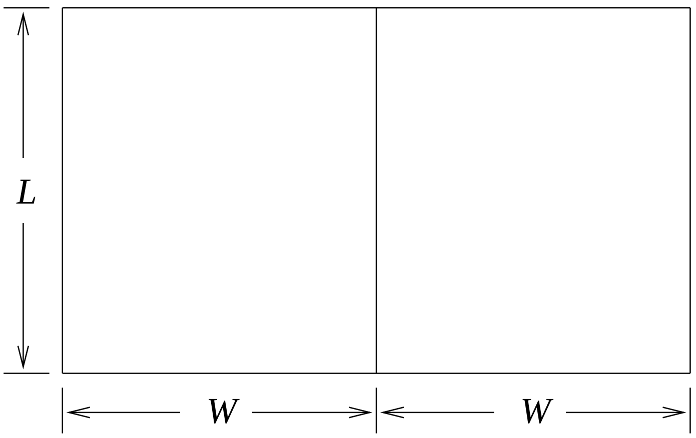
c) On what intervals is h concave up? On what intervals is h concave down? Briefly explain your assertions.

d) For which values of x, if any, does h have relative maxima on $[1, 6]$? For which values of x, if any, does h have relative mimina on $[1, 6]$? For which values of x, if any, does h have inflection points on $[1, 6]$?

e) Use the information you have obtained in previous parts of this problem to sketch a possible graph of $y = h(x)$ using the axis below. Label any relative extrema of h and any inflection points of h on the graph you sketch.



(14) 9. A rectangular field is to have area 600 square feet. Fencing is required to enclose the field and to divide it in half. Find L and W as shown below so that the field requires the smallest total fence length. Explain briefly why your answer gives a minimum.



(10) 10. Suppose you know that $f(5) = 7$, $f'(5) = 2$, $g(7) = 3$, and $g'(7) = 8$.

a) If $F(x) = g(f(x))$, compute $F(5)$ and $F'(5)$.

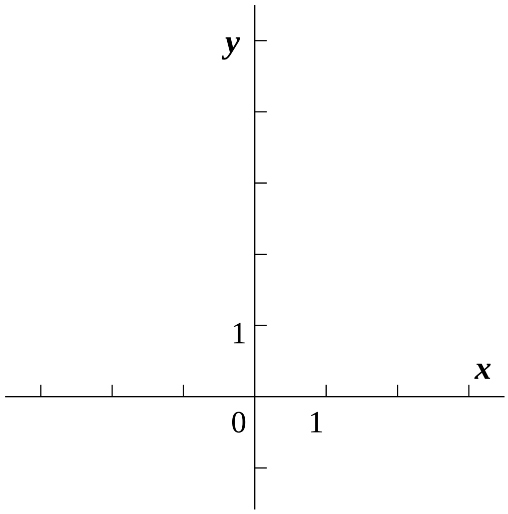
b) Use linear approximation or differentials to give an approximate value for $F(5.02)$.

(12) 11. a) Suppose $G(x) = (x \ln x) - x$. Compute $G'(x)$.

b) Use your answer to a) to evaluate $\displaystyle\int_1^{e^2} \ln x \, dx$. Computations with exp and log should be simplified as much as possible. A numerical approximation like 1.40135 is **not** acceptable.

(8) 12. Suppose $k''(x) = -6x^2$ and $k(1) = 3$ and $k'(1) = 5$. Find a formula for $k(x)$.

(14) 13. Sketch the region in the plane bounded by the $x-$axis and the graph of $y = 4 - x^2$. Then find the area of this region using methods of calculus.

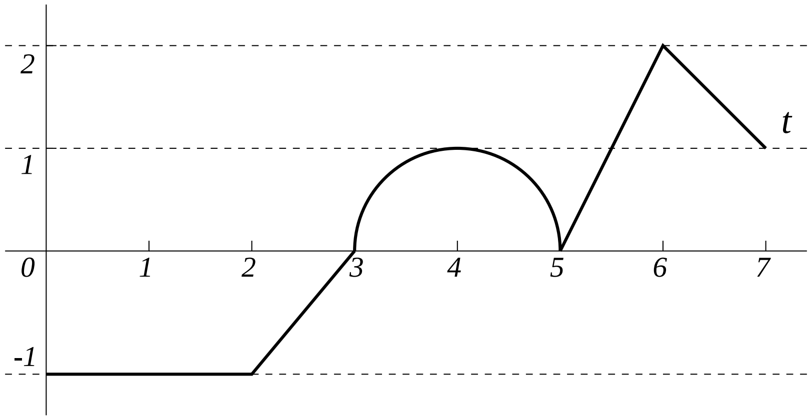


(16) 14. Evaluate the indicated integrals using methods of calculus:

a) $\displaystyle\int_1^2 \left(2\sqrt{x} - \frac{5}{x^2}\right) dx$

b) $\displaystyle\int (\sin 5x)^2 \cos 5x \, dx$

(12) 15. Below is a graph of the function H which is defined on the interval $[0, 7]$. The graph is made up of pieces of line segments, except on the interval $[3, 5]$, where it is a semicircle.



The graph of H

a) Compute $\displaystyle\int_1^4 H(t)\,dt$.

b) If Q is the function defined by $Q(x) = \displaystyle\int_0^x H(t)\,dt$, what is $Q'(4)$?

c) Compute the Riemann sum estimate for $\displaystyle\int_2^7 H(t)\,dt$ associated with the partition $2 < 3 < 4 < 6 < 7$ where the sample points are the right endpoints of the subintervals.

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Math 135 Final Exam

Thursday, December 18, 1997, from 4 PM to 7 PM

NAME (*please print*): ______________________________

SIGNATURE: ______________________________

SECTION #: __________ LECTURER: ____________________

Do all problems, in any order.

Show all your work. Full credit may not be given for an answer alone.

You may use one sheet of notes and any standard calculator without a QWERTY keypad on this exam. You may use no other materials.

Problem Number	Possible Points	Points Earned:
1	8	
2	12	
3	12	
4	14	
5	12	
6	10	
7	20	
8	26	
9	14	
10	10	
11	12	
12	8	
13	14	
14	16	
15	12	
Total Points Earned:		

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