1 B

(17) 1. Find $\dfrac{dy}{dx}$:

a) $y = 3e^{2x} - 5x\ln(x^2+1)$

b) $y = \sin\left(\dfrac{x^2+3}{x^2-1}\right)$

c) $y^2 - x^3y + 2xy^5 = 4$

(17) 2. In this problem the function G is defined by the formula

$$G(x) = (2 - e^x)(1 - e^x)$$

a) What is $\lim_{x \to -\infty} G(x)$? What is $\lim_{x \to +\infty} G(x)$?

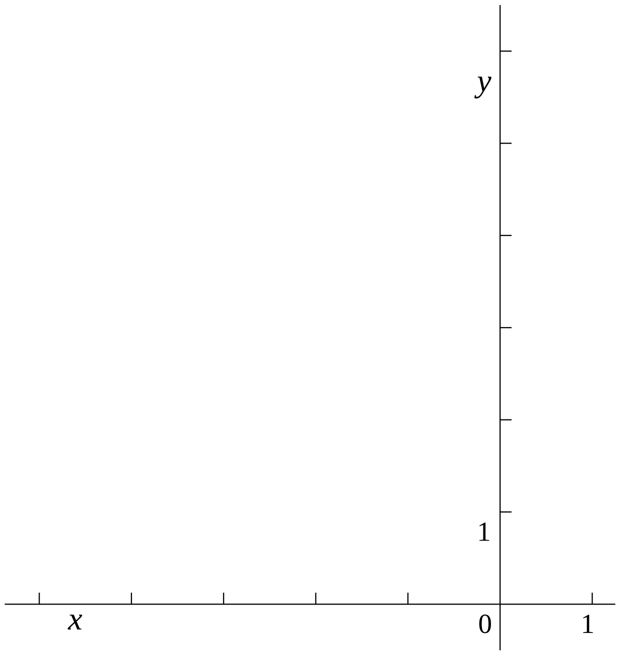
b) For which x is $G(x) = 0$? Computations with exp and log should be simplified as much as possible. A numerical approximation like 1.40135 is **not** acceptable.

c) Compute $G'(x)$. For which x is $G'(x) = 0$? Computations with exp and log should be simplified as much as possible. A numerical approximation like 1.40135 is **not** acceptable.

THIS PROBLEM CONTINUES ON THE NEXT PAGE.

d) Compute $G''(x)$. For which x is $G''(x) = 0$? Computations with exp and log should be simplified as much as possible. A numerical approximation like 1.40135 is **not** acceptable.

e) Sketch a graph of $y = G(x)$, using the information you have obtained in the previous parts of this problem. Indicate precisely any relative maxima or minima or points of inflection with exact coordinates, and any vertical or horizontal asymptotes with precise equations. Computations with exp and log should be simplified as much as possible. A numerical approximation like 1.40135 is **not** acceptable.



(16) 3. A rectangular box with *no* top has height H and a square base with sides each S in length. Its volume is $\frac{4}{27}$ cubic feet. Find the specific values of S and H so that the box will have the *least* total surface area. Explain briefly why you have found the box with the least surface area.

(22) 4. Suppose that $F'(x)$, **the derivative** of F, is given by the formula

$$F'(x) = x^{30}(x-1)^{57}$$

Do **not** try to find a formula for F itself! That is difficult to do and will not help you complete this problem.

a) For which x is $F'(x) = 0$? For which x is $F'(x) > 0$? For which x is $F'(x) < 0$?

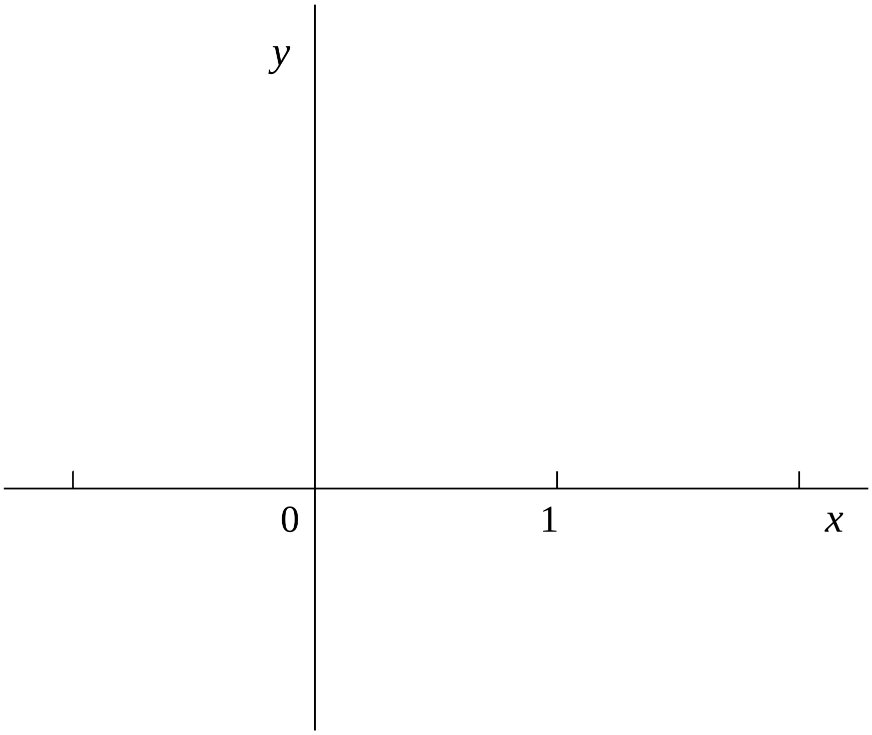
b) What are the critical numbers of F? Which of these are relative maxima and which are relative minima and which are neither? Give evidence to support your assertions.

c) Find a formula for $F''(x)$. For which x is $F''(x) = 0$? For which x is $F''(x) > 0$? For which x is $F''(x) < 0$?

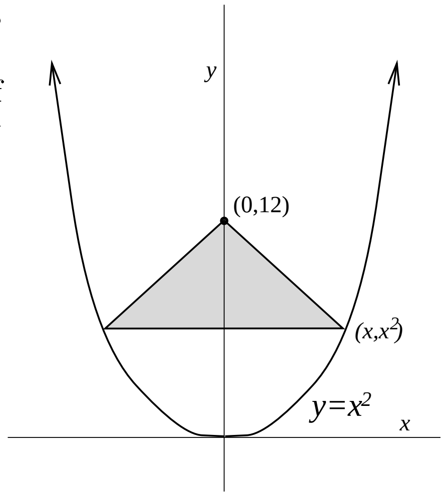
THIS PROBLEM CONTINUES ON THE NEXT PAGE.

d) What are the points of inflection of F? Why are they points of inflection? Give evidence to support your assertions.

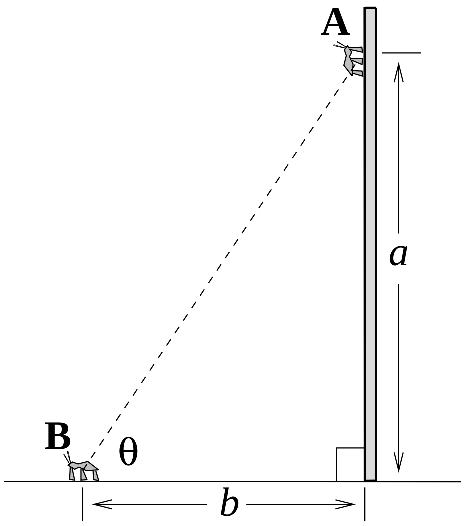
e) Use the information you have obtained in earlier parts of this problem to sketch a possible graph of $y = F(x)$. Make sure that your graph is qualitatively correct and uses all the information you have deduced in previous parts of this problem. Label any points of inflection, relative maxima, and relative minima.



(15) 5. A triangle has one vertex at the point $(0,12)$ and its opposite side, below it, is parallel to the $x-$axis with endpoints on the parabola $y=x^2$. Find all of the vertices of such a triangle which has the *largest* area. Explain briefly why you have found the largest such triangle.



(13) 6. Ant **A** is crawling up a vertical pole at 0.3 feet per minute. At the same time ant **B** is crawling away from the base of the pole on the horizontal ground at 0.4 feet per minute.



a) If θ is the angle that ant **B** sees between the base of the pole and ant **A**, if a is the distance from ant **A** to the base of the pole, and if b is the distance from ant **B** to the base of the pole, write an equation connecting θ with a and b.

You will need to use a trigonometric function.

b) Use the information provided to compute θ and $\dfrac{d\theta}{dt}$ at the instant that ant **A** is 10 feet up the pole and ant **B** is 5 feet from the base of the pole. Is the angle θ increasing or decreasing at this time?

B B

Exam 2 for Math 135

Sections 16, 17, and 18

November 17, 1997

NAME (*please print*): ____________________

SIGNATURE: ____________________

SECTION #: ____________________

Do all problems, in any order.

Show all your work. Full credit may not be given for an answer alone.

You may use one sheet of notes and any standard calculator without a QWERTY keypad on this exam. You may use no other materials.

Problem Number	Possible Points	Points Earned:
1	17	
2	17	
3	16	
4	22	
5	15	
6	13	
Total Points Earned:		

B B