(15) 1. If A is real and positive, and $A \neq 1$, `Maple` reports that

$$\int_{-\infty}^{\infty} \frac{x^2\,dx}{(x^2+1)(x^2+A^2)} = \frac{\pi}{A+1}.$$

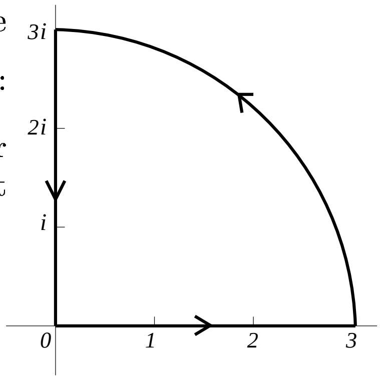
Check this assertion using the Residue Theorem.
Sketch the contour of integration.
Show any residue computations.
Explain why some integral has limiting value equal to 0.

For a rather small amount of extra credit* please *guess* the value of $\int_{-\infty}^{\infty} \frac{x^2\,dx}{(x^2+1)^2}$.

Answer ______________

(14) 2. a) State a version of the Cauchy integral formula for derivatives**.

b) Use the statement in a) or some other result to compute $\int_B \frac{z^4}{\big(z-(1+i)\big)^3}\,dz$ where B is the simple closed curve shown: the line segment from 0 to 3, followed by the quarter-circular arc centered at 0 from 3 to $3i$, followed by the line segment from $3i$ to 0.



Answer -24π

(15) 3. In this problem, $f(z) = \frac{1}{e^z-1} - \frac{1}{z}$. Find and classify (as removable, pole, or essential) all isolated singularities of $f(z)$. Find the residue of $f(z)$ at *every* singularity. For each singularity which is a pole, find the order of the pole.

Comment This is a *complex* analysis course! Please find **all** isolated singularities!

(12) 4. a) Suppose that $f(z)$ is an entire function and there is a positive constant K so that $|f(z)| > K$ for all z. Prove that $f(z)$ must be a constant function.

Hint What can you do with something that is *not* 0?

b) The exponential function is never 0 and is an entire function. Briefly explain why the exponential function does not contradict the assertion in part a).

(14) 5. Find the first four non-zero terms of the Taylor series centered at $z=0$ for the function $f(z) = \frac{\sin z}{(z-1)^2}$.

Suggestion A direct computation *is* possible. The results are messy, with a large chance for error. Try another way.

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* Let's say, uhhh, 3 points.

** Your statement should contain the words "simply" and "simple", and be valid for derivatives of any order.

(15) 6. In this problem, $f(z) = \dfrac{\sqrt{z}}{(z-1)^2}$.

a) Specify precisely a maximal (largest) domain in $\mathbb{C}$ in which $f(z)$ is analytic. (You need *not* prove your assertion!)

b) What is the radius of convergence of the Taylor series centered at $z = 1 + i$ for $f(z)$?

Note The coefficients of this series are complicated. You may use your answer to a) here.

Answer _______________

c) What is the radius of convergence of the Taylor series centered at $z = -2 + i$ for $f(z)$?

Note The coefficients of this series are complicated. The answer is *tricky*.

Answer _______________

d) Find the first four non-zero terms of the Laurent series centered at $z = 1$ for $f(z)$.

(15) 7. Suppose a and b are real and positive, and $a > b$. Use the Residue Theorem or other results of this course to compute

$$\int_0^{2\pi} \frac{d\theta}{a + b\cos\theta}.$$

Note This is problem #7 of section 2.3 in your text. The answer given in the text is $\dfrac{\pi}{\sqrt{a^2 - b^2}}$ which is slightly (?) incorrect.

Second Exam for Math 403, section 2

April 19, 2005

NAME ______________________________

Do all problems, in any order.
Show your work. An answer alone may not receive full credit.
No notes, texts, or calculators may be used on this exam.

Problem Number	Possible Points	Points Earned:
1	15	
2	14	
3	15	
4	12	
5	14	
6	15	
7	15	
Total Points Earned:		