

AB 1

Suppose L_1 is described parametrically by $c_1(t) = (3t + 1, 7t + 2, -t + 1)$ and L_2 is described parametrically by $c_2(s) = (2s - 2, s + 6, -2s + 6)$.

- a) These lines intersect each other when their x,y, and z components are equal. That is...

$$x) \quad 3t + 1 = 2s - 2 \quad \Rightarrow \quad s = (3/2)(t + 1)$$

$$y) \quad 7t + 2 = s + 6 \quad \Rightarrow \quad 7t + 2 = (3/2)(t + 1) + 6 \quad \Rightarrow \quad t = 1$$

$$z) \quad -t + 1 = -2s + 6 \quad \Rightarrow \quad -(1) + 1 = -2s + 6 \quad \Rightarrow \quad s = 3$$

Plugging $(t = 1)$ into $c_1(t)$ yields $(4, 9, 0)$. Similarly, plugging $(s = 3)$ into $c_2(s)$ yields $(4, 9, 0)$. Therefore, we find that $c_1(t)$ and $c_2(s)$ intersect at $(4, 9, 0)$.

- b) The direction vectors of the parameterizations $c_1(t)$ and $c_2(s)$ are, respectively, $\langle 3, 7, -1 \rangle$ and $\langle 2, 1, -2 \rangle$. By taking the cross product of these two vectors we get a normal vector perpendicular to both L_1 and L_2 . So...

$$\langle 3, 7, -1 \rangle \times \langle 2, 1, -2 \rangle = \det \begin{array}{ccc} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 7 & -1 \\ 2 & 1 & -2 \end{array}$$

$$= (7(-2) - 1(-1)) \cdot \mathbf{i} - (3(-2) - 2(-1)) \cdot \mathbf{j} + (3(1) - 7(2)) \cdot \mathbf{k} \\ = 13 \cdot \mathbf{i} + 6 \cdot \mathbf{j} - 11 \cdot \mathbf{k} = \langle 13, 6, -11 \rangle .$$

Therefore, we have a point on both lines, $(4, 9, 0)$, and a normal vector, $\langle 13, 6, -11 \rangle$, so our equation for the plane containing L_1 and L_2 is...

$$13 \cdot (x - 4) + 6 \cdot (y - 9) - 11 \cdot (z - 0) = 0, \text{ or}$$

$$13x + 6y - 11z = 88$$