(8) 1. If $z = 5y + f(3x^2 - 7y^2)$, where f is differentiable, show that $7y\frac{\partial z}{\partial x} + 3x\frac{\partial z}{\partial y} = 15x$.

A variation of #39 on p. 976

(12) 2. The average value of a function f defined in a region R of $\mathbb{R}^2$ is $\dfrac{\int\int_R f\,dA}{\int\int_R dA}$.

Compute the average distance to the origin of points in the right half plane ($x \geq 0$) which are inside the unit circle.

Resembling review problem **W**

(12) 3. Leonhard Euler (1707–1783) was a great and very prolific mathematician. He published *Institutiones Calculi Differentialis* (*Methods of the Differential Calculus*) in 1755. It was an influential text, and was the first source of criteria for discovering local extrema of functions of several variables. In it Euler investigated the following specific example: $V = x^3 + y^2 - 3xy + \frac{3}{2}x$.
He asserted that V has a minimum both at $\left(1, \frac{3}{2}\right)$ and at $\left(\frac{1}{2}, \frac{3}{4}\right)$. Was Euler correct?

Reference: *A History of Mathematics* by Victor J. Katz, Harper Collins, 1993, p. 517.

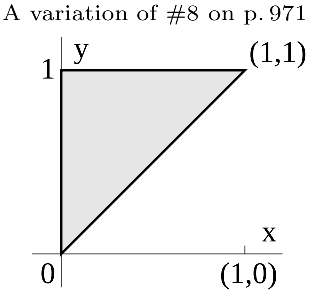
You must check that Euler found all the critical points and only the critical points, and also that he classified them correctly.

Resembling review problem **E** and many other problems

(12) 4. Use Lagrange multipliers to find the maximum and minimum values of the function subject to the given constraint: $f(x, y, z) = 8x - 4z$; $\quad x^2 + 6y^2 + 3z^2 = 5$.

A variation of #8 on p. 971

(10) 5. Compute $\int\int_R e^{3x/y}\,dA$ where the region R is the triangle shown.



Like many problems in section 15.2

(16) 6. Evaluate the triple integral $\int\int\int_E 7xy\,dV$ where E lies under the plane $z = 1 + x + y$ and above the region in the xy-plane bounded by the curves $y = \sqrt{x}$, $y = 0$, and $x = 1$.

A variation of #9 on p. 1030

(12) 7. Consider the iterated triple integral $\int_0^{\pi/2}\int_0^{\pi/2}\int_1^2 \rho^2 \sin\phi\,d\rho\,d\phi\,d\theta$.
a) Compute the integral.

b) The integral evaluates the volume of a solid in $\mathbb{R}^3$ with spherical coordinates. Use complete English sentences possibly accompanied by a sketch to describe this solid.

Essentially #12 on p. 1050

(18) 8. a) Compute $\int_C (x^2 + y)\,dx + (-x+3)\,dy$ if C is the curve $y = x^3$ from $x = 0$ to $x = 2$.

b) Suppose $\mathbf{V}$ is the vector field defined by $\mathbf{V}(x,y) = (x^2+y)\mathbf{i} + (x+3)\mathbf{j}$. Find a scalar function $f(x,y)$ defined everywhere in the plane so that $\nabla f = \mathbf{V}$. You must show a process leading from $\mathbf{V}$ to f. Compute $\int_C \mathbf{V}\cdot\mathbf{T}\,ds$ if C is the curve $\begin{cases} x(t) = 5\cos t - 2(\cos t)^4 \\ y(t) = 4 - 2(\sin t)^2 \end{cases}$ for t in the interval $\left[0, \frac{\pi}{2}\right]$. Here $\mathbf{T}$ is the unit tangent vector to the curve, C.

Resembling several review problems and many textbook problems

A A

Second Exam for Math 251, sections 5–10

April 7, 2006

NAME ____

Please circle your section number: 5 6 7 8 9 10

Do all problems, in any order.
Show your work. An answer alone may not receive full credit.
No notes other than the distributed formula sheet may be used on this exam.
No calculators may be used on this exam.

Problem Number	Possible Points	Points Earned:
1	8	
2	12	
3	12	
4	12	
5	10	
6	16	
7	12	
8	18	
Total Points Earned:		

A A