Math 152, Spring 2008, Review Problems for the Final Exam

Your final exam is likely to have problems that do not resemble these review problems. You should also look at the review problems for the first and second exams.

(1) Let $\mathcal{R}$ be the region in the xy-plane bounded by $y = x^2 - 4x + 4$ and $y = x$. (a) Find the volume of the solid obtained by rotating $\mathcal{R}$ about the x-axis. (b) Find the volume of the solid obtained by rotating $\mathcal{R}$ about the y-axis.

(2) Evaluate $\displaystyle\int \frac{x^2}{1-x^2}\,dx$ and $\displaystyle\int \frac{e^x}{e^{2x}+4}\,dx$.

(3) Find a reduction formula for the integral $\displaystyle\int_2^\infty x^n e^{-5x}\,dx$.

(4) Find a reduction formula for $\displaystyle\int \tan^n x\,dx$.

(5) Find $\displaystyle\int \frac{dx}{x^2+10x+34}$ and $\displaystyle\int \frac{dx}{\sqrt{2x-x^2}}$.

(6) Find $\displaystyle\int \frac{dx}{(1+x^2)^3}$ and $\displaystyle\int \sin^5 x\cos^2 x\,dx$.

(7) Find N such that the Trapezoidal Rule with N subintervals approximates $\displaystyle\int_3^5 e^{-x^2}\,dx$ with accuracy better than 0.0001.

(8) Find $\displaystyle\int x(\ln x)^2\,dx$ and $\displaystyle\int \tan^{-1} x\,dx$.

(9) Find the surface area of the surface obtained by rotating $y = \sqrt{7-x^2}$, $0 \le x \le 1$ about the x-axis.

(10) Find the solution of $\dfrac{dy}{dx} = 1 + y^2$ with initial condition $y(2) = 0$.

(11) A student writes $\displaystyle\sum_{n=1}^{\infty}[n-(n+1)] = [1-2]+[2-3]+[3-4]+[4-5]+\cdots$ and concludes that this sum equals 1 because all the other terms cancel out in the telescoping series. Use partial sums to explain why this conclusion is not correct.

(12) In 1746, the great mathematician Euler published a journal article with the following computation: $\displaystyle\sum_{n=0}^{\infty} x^n + \sum_{n=1}^{\infty} x^{-n} = \frac{1}{1-x} + \sum_{n=1}^{\infty}(1/x)^n = \frac{1}{1-x} + \frac{1/x}{1-1/x} = 0$. Explain why this computation is **wrong** for **every** value of x. Later in the same article, Euler warned the reader about making mistakes of this nature. Maybe Euler wanted to see if anyone would catch the error.

(13) Find the Maclaurin series of $(x - \sin x)/x^3$.

(14) Find the interval of convergence of $\sum_{n=1}^{\infty} 2^n (x-3)^n$.

(15) The Maclaurin series of $\tan x$ begins with $b_1 x + b_3 x^3 + b_5 x^5$. Find b_1, b_3, b_5 using the fact $(\cos x)(\tan x) = \sin x$ and the known Maclaurin series of $\cos x$ and $\sin x$.

(16) For each series below, determine whether it converges of diverges. Show how the various tests are used.

$$\sum_{n=1}^{\infty} \frac{n^5}{(1.1)^n} \qquad \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\ln(n+4)} \qquad \sum_{n=4}^{\infty} \frac{1}{n(\ln n)(\ln(\ln n))} \qquad \sum_{n=1}^{\infty} \frac{n^2+n+3}{n^4+n^3+n+4} \qquad \sum_{n=1}^{\infty} \frac{(-1)^n n}{n+4}\,.$$

(17) Find a value of N such that $\displaystyle\sum_{n=1}^{N} \frac{(-1)^n}{\sqrt{n}}$ approximates $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ with an accuracy better than 10^{-6}.

(18) Express the parametric equation $x = \tan t$, $y = \cos^2 t$ in the form $y = f(x)$ by eliminating the parameter.

(19) Consider the parametric curve given by the equations $x = t^3 + 1$, $y = t^4 - 2$. Find a value of t for which we have $\dfrac{dy}{dx} = 1$.

(20) Find the length of the curve given parametrically by $x = 5 + \sin(4t)$, $y = 3 + \cos(4t)$ for $0 \le t \le \pi/8$.

(21) Show that the polar equation $r = 8\sin\theta$ gives us a circle in the xy-plane.