Formula Sheets for the Math 152 Final Exam

The solutions of $ax^2+bx+c=0$ are $x=\big(-b\pm\sqrt{b^2-4ac}\big)/(2a)$.

$$e^{a+b}=e^ae^b\ ,\quad \ln(ab)=(\ln a)+(\ln b)\ ,\quad \ln(a^b)=b(\ln a)\ ,\quad \ln(1)=0\ ,\quad \ln(e)=1$$

$$e^{\ln x}=x\ ,\quad \ln(e^x)=x\ ,\quad \tfrac{d}{dx}(e^x)=e^x\ ,\quad \tfrac{d}{dx}(\ln x)=1/x\ ,\quad \int\tfrac{du}{u}=\ln|u|+C$$

$$\sin(0)=0\ ,\ \sin(\pi/6)=1/2\ ,\ \sin(\pi/4)=\sqrt{2}/2\ ,\ \sin(\pi/3)=\sqrt{3}/2\ ,\ \sin(\pi/2)=1$$

$$\cos(0)=1\ ,\ \cos(\pi/6)=\sqrt{3}/2\ ,\ \cos(\pi/4)=\sqrt{2}/2\ ,\ \cos(\pi/3)=1/2\ ,\ \cos(\pi/2)=0$$

$$\tan x=\sin x/\cos x\ ,\quad \cot x=\cos x/\sin x\ ,\quad \sec x=1/\cos x\ ,\quad \csc x=1/\sin x$$

$$\cos^2 x+\sin^2 x=1\ ,\quad 1+\tan^2 x=\sec^2 x\ ,\quad 1+\cot^2 x=\csc^2 x$$

$$\sin A\cos B=(1/2)[\sin(A-B)+\sin(A+B)]$$

$$\sin A\sin B=(1/2)[\cos(A-B)-\cos(A+B)]$$

$$\cos A\cos B=(1/2)[\cos(A-B)+\cos(A+B)]$$

$$\sin(2x)=2\sin x\cos x\ ,\quad \cos(2x)=\cos^2 x-\sin^2 x$$

$$\cos^2 x=(1/2)\big(1+\cos(2x)\big)\ ,\quad \sin^2 x=(1/2)\big(1-\cos(2x)\big)$$

$$\tfrac{d}{dx}(\sin x)=\cos x\ ,\quad \tfrac{d}{dx}(\tan x)=\sec^2 x\ ,\quad \tfrac{d}{dx}(\sec x)=\sec x\tan x$$

$$\tfrac{d}{dx}(\cos x)=-\sin x\ ,\quad \tfrac{d}{dx}(\cot x)=-\csc^2 x\ ,\quad \tfrac{d}{dx}(\csc x)=-\csc x\cot x$$

$$\frac{d}{dx}(\sin^{-1} x)=\frac{1}{\sqrt{1-x^2}}\ ,\quad \frac{d}{dx}(\tan^{-1} x)=\frac{1}{1+x^2}\ ,\quad \frac{d}{dx}(\sec^{-1} x)=\frac{1}{x\sqrt{x^2-1}}$$

$$\int\sec x\,dx=\ln|\sec x+\tan x|+C\ ,\quad \int\csc x\,dx=\ln|\csc x-\cot x|+C$$

The area between concentric circles is $\pi(\text{outer radius})^2-\pi(\text{inner radius})^2$.

The area of a cylinder is $(2\pi\,\text{radius})(\text{height})$.

If the force is constant then work = force × distance.

The average value of f on $[a,b]$ is $\frac{1}{b-a}\int_a^b f(x)\,dx$.

Midpoint Rule: $\Delta x[f(c_1)+f(c_2)+\cdots+f(c_N)]$ where $c_j=(x_{j-1}+x_j)/2$. Trapezoidal Rule: $\dfrac{\Delta x}{2}[f(x_0)+2f(x_1)+2f(x_2)+\cdots+2f(x_{n-2})+2f(x_{n-1})+f(x_N)]$. Simpson's Rule: $\dfrac{\Delta x}{3}[f(x_0)+4f(x_1)+2f(x_2)+4f(x_3)+2f(x_4)+\cdots+2f(x_{n-2})+4f(x_{n-1})+f(x_N)]$.

Sometimes, we write error = approximation − exact value. If error(T_N) and error(M_N) are the errors for the Trapezoidal Rule and Midpoint Rule, respectively, then

$$|\text{error}(T_N)|\le\frac{K_2(b-a)^3}{12N^2}\ \text{and}\ |\text{error}(M_N)|\le\frac{K_2(b-a)^3}{24N^2},\ \text{if}\ |f''(x)|\le K_2\ \text{for}\ a\le x\le b.$$

If error(S_N) is the error for Simpson's Rule, then

$$|\text{error}(S_N)|\le\frac{K_4(b-a)^5}{180N^4}\ \text{where}\ |f^{(4)}(x)|\le K_4\ \text{for}\ a\le x\le b.$$

If a rational function is proper and has $(x-a)^M$ in the denominator, then the partial fraction expansion must include $\dfrac{A_1}{x-a}+\dfrac{A_2}{(x-a)^2}+\cdots+\dfrac{A_M}{(x-a)^M}$. If $b>0$ and a proper rational function has $(x^2+b)^N$ in the denominator, then the partial fraction expansion must include $\dfrac{A_1x+B_1}{x^2+b}+\dfrac{A_2x+B_2}{(x^2+b)^2}+\cdots+\dfrac{A_Nx+B_N}{(x^2+b)^N}$.

length $= \int_a^b \sqrt{1+[f'(x)]^2}\,dx$, surface area $= 2\pi \int_a^b f(x)\sqrt{1+[f'(x)]^2}\,dx$

The nth Taylor polynomial of $f(x)$ with center c is $T_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(c)}{k!}(x-c)^k$. If $|f^{(n+1)}(u)| \le K$ for all u between c and x, then $|f(x) - T_n(x)| \le K\frac{|x-c|^{n+1}}{(n+1)!}$.

$$\lim_{n\to\infty} n^{1/n} = 1 \ ; \quad \lim_{n\to\infty} \frac{x^n}{n!} = 0 \ ; \quad \lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n = e^x \ ; \quad \lim_{n\to\infty}\frac{n^k}{a^n} = 0 \text{ if } a > 1.$$

$\lim_{n\to\infty} r^n = 0$ when $|r| < 1$; $\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}$ when $|r| < 1$.

$\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if $p > 1$ (and diverges if $p \le 1$).

If the statement $\lim_{n\to\infty} a_n = 0$ is false, then $\sum_{n=1}^{\infty} a_n$ diverges.

If $f(x)$ is a positive decreasing continuous function on $[N, \infty)$ and $a_n = f(n)$ then
$\int_{n+1}^{\infty} f(x)\,dx \le a_{n+1} + a_{n+2} + a_{n+3} + \cdots \le \int_n^{\infty} f(x)\,dx$.
In addition, $\sum_{n=N}^{\infty} a_n$ and $\int_N^{\infty} f(x)\,dx$ both converge or both diverge.

If M is a natural number and $0 \le a_n \le b_n$ for $n \ge M$ then: (a) If $\sum_{n=1}^{\infty} b_n$ converges then $\sum_{n=1}^{\infty} a_n$ converges, (b) if $\sum_{n=1}^{\infty} a_n$ diverges then $\sum_{n=1}^{\infty} b_n$ diverges.

Assume $a_n > 0$, $b_n > 0$, $\lim_{n\to\infty} \frac{a_n}{b_n} = L$. If $0 < L < \infty$ then $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ both converge or both diverge. If $L = 0$ and $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.

If $a_n > 0$, $a_1 \ge a_2 \ge a_3 \ge \cdots$ and $\lim\limits_{n\to\infty} a_n = 0$ then $\sum_{n=1}^{\infty}(-1)^n a_n$ converges.

$\sum a_n$ converges absolutely when $\sum |a_n|$ converges. $\sum a_n$ converges conditionally when it converges, but does not converge absolutely. If $\sum a_n$ converges absolutely, then $\sum a_n$ converges.

$$\text{If } a_n \ne 0 \text{ and } \lim_{n\to\infty}\frac{|a_{n+1}|}{|a_n|} = \rho \text{ then } \begin{cases} \sum a_n \text{ converges absolutely if } \rho < 1, \\ \sum a_n \text{ diverges if } \rho > 1, \\ \text{the test is inconclusive if } \rho = 1. \end{cases}$$

$$\text{If } \lim_{n\to\infty} |a_n|^{1/n} = L \text{ then } \begin{cases} \sum a_n \text{ converges absolutely if } L < 1, \\ \sum a_n \text{ diverges if } L > 1, \\ \text{the test is inconclusive if } L = 1. \end{cases}$$

$$\text{The Taylor series of } f(x) \text{ with center } c \text{ is } \sum_{n=0}^{\infty} \frac{f^{(n)}(c)}{n!}(x-c)^n.$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \ ; \quad \sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} \ ; \quad \cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} \ ;$$

$$(1+x)^a = 1 + \sum_{n=1}^{\infty}\left(\frac{a(a-1)(a-2)\cdots(a-n+1)}{n!}\right)x^n \text{ if } |x| < 1.$$

$$\text{length } = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt, \ \text{length } = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\,d\theta, \ \text{area } = \int_a^b \frac{r^2}{2}\,d\theta$$