Formula Sheet for Math 152, Exam 1

The solutions of $ax^2 + bx + c = 0$ are $x = \big(-b \pm \sqrt{b^2 - 4ac}\big)/(2a)$.

$$e^{a+b} = e^a e^b \ , \quad \ln(ab) = (\ln a) + (\ln b) \ , \quad \ln(a^b) = b(\ln a) \ , \quad \ln(1) = 0 \ , \quad \ln(e) = 1$$

$$e^{\ln x} = x \ , \quad \ln(e^x) = x \ , \quad \tfrac{d}{dx}(e^x) = e^x \ , \quad \tfrac{d}{dx}(\ln x) = 1/x \ , \quad \int \tfrac{du}{u} = \ln|u| + C$$

$$\sin(0) = 0 \ , \ \sin(\pi/6) = 1/2 \ , \ \sin(\pi/4) = \sqrt{2}/2 \ , \ \sin(\pi/3) = \sqrt{3}/2 \ , \ \sin(\pi/2) = 1$$

$$\cos(0) = 1 \ , \ \cos(\pi/6) = \sqrt{3}/2 \ , \ \cos(\pi/4) = \sqrt{2}/2 \ , \ \cos(\pi/3) = 1/2 \ , \ \cos(\pi/2) = 0$$

$$\tan x = \sin x / \cos x \ , \quad \cot x = \cos x / \sin x \ , \quad \sec x = 1/\cos x \ , \quad \csc x = 1/\sin x$$

$$\cos^2 x + \sin^2 x = 1 \ , \quad 1 + \tan^2 x = \sec^2 x \ , \quad 1 + \cot^2 x = \csc^2 x$$

$$\sin A \cos B = (1/2)[\sin(A - B) + \sin(A + B)]$$

$$\sin A \sin B = (1/2)[\cos(A - B) - \cos(A + B)]$$

$$\cos A \cos B = (1/2)[\cos(A - B) + \cos(A + B)]$$

$$\sin(2x) = 2 \sin x \cos x \ , \quad \cos(2x) = \cos^2 x - \sin^2 x$$

$$\cos^2 x = (1/2)\big(1 + \cos(2x)\big) \ , \quad \sin^2 x = (1/2)\big(1 - \cos(2x)\big)$$

$$\tfrac{d}{dx}(\sin x) = \cos x \ , \quad \tfrac{d}{dx}(\tan x) = \sec^2 x \ , \quad \tfrac{d}{dx}(\sec x) = \sec x \tan x$$

$$\tfrac{d}{dx}(\cos x) = -\sin x \ , \quad \tfrac{d}{dx}(\cot x) = -\csc^2 x \ , \quad \tfrac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1 - x^2}} \ , \quad \frac{d}{dx}(\tan^{-1} x) = \frac{1}{1 + x^2} \ , \quad \frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2 - 1}}$$

$$\int \sec x \, dx = \ln|\sec x + \tan x| + C \ , \quad \int \csc x \, dx = \ln|\csc x - \cot x| + C$$

The area between concentric circles is $\pi(\text{outer radius})^2 - \pi(\text{inner radius})^2$.

The area of a cylinder is $(2\pi \, \text{radius})(\text{height})$.

If the force is constant then work = force $\times$ distance.

The average value of f on $[a, b]$ is $\frac{1}{b-a}\int_a^b f(x)\, dx$.

Midpoint Rule: $\Delta x[f(c_1) + f(c_2) + \cdots + f(c_N)]$ where $c_j = (x_{j-1} + x_j)/2$. Trapezoidal Rule: $\dfrac{\Delta x}{2}[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-2}) + 2f(x_{n-1}) + f(x_N)]$. Simpson's Rule: $\dfrac{\Delta x}{3}[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_N)]$.

If Error(T_N) and Error(M_N) are the errors for the Trapezoidal Rule and Midpoint Rule, respectively, then

$$|\text{Error}(T_N)| \le \frac{K_2(b-a)^3}{12N^2} \text{ and } |\text{Error}(M_N)| \le \frac{K_2(b-a)^3}{24N^2}, \text{ if } |f''(x)| \le K_2 \text{ for } a \le x \le b.$$

If Error(S_N) is the error for Simpson's Rule, then

$$|\text{Error}(S_N)| \le \frac{K_4(b-a)^5}{180N^4} \text{ if } |f^{(4)}(x)| \le K_4 \text{ for } a \le x \le b.$$

If a rational function is proper and has $(x - a)^M$ in the denominator, then the partial fraction expansion must include $\dfrac{A_1}{x - a} + \dfrac{A_2}{(x - a)^2} + \cdots + \dfrac{A_M}{(x - a)^M}$. If $b > 0$ and a proper rational function has $(x^2 + b)^N$ in the denominator, then the partial fraction expansion must include $\dfrac{A_1 x + B_1}{x^2 + b} + \dfrac{A_2 x + B_2}{(x^2 + b)^2} + \cdots + \dfrac{A_N x + B_N}{(x^2 + b)^N}$.