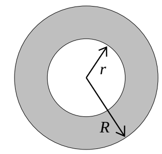
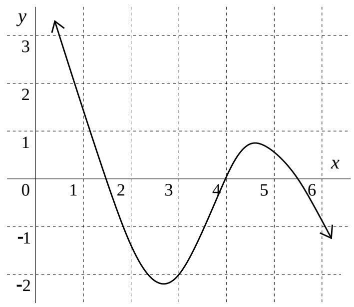
Some answers to review questions for the second exam for Math 151

0. Two circles have the same center. The inner circle has radius r which is increasing at the rate of 3 inches per second. The outer circle has radius R which is increasing at the rate of 2 inches per second. Suppose that A is the area of the region *between* the circles. At a certain time, r is 7 inches and R is 10 inches. What is A at that time? How fast is A changing at that time? Is A increasing or decreasing at that time?



Answer $A = \pi R^2 - \pi r^2$. If $R = 10$ and $r = 7$, $A = \pi(10^2) - \pi(7^2) = 51\pi$. If we d/dt the equation, the result is $A' = 2\pi R\frac{dR}{dt} - 2\pi r\frac{dr}{dt} = 2\pi(10)(2) - 2\pi(7)(3) = -2\pi$. The area is changing at 2π inches per second and it is *decreasing.*

8. The graph of $y = f'(x)$, the *derivative* of the function $f(x)$, is shown to the right. Use the graph to answer the questions below.



The parts of this problem are *not* related but both parts use information from the graph of the derivative of $f'(x)$.

a) Use information from the graph of $f'(x)$ to find (as well as possible) the x where the *maximum value* of $f(x)$ in the interval $1 \leq x \leq 3$ will occur. Briefly explain using calculus why your answer is correct, including verification that the value of $f(x)$ you select is larger than $f(x)$ at *any* other number in the interval. **Answer** x is ≈ 1.6, the x−intercept of $y = f'(x)$ between 1 and 2. Call this x^*. $f'(x) < 0$ between x^* and 3, so the function decreases from x^* to 3: $f(x^*) > f(x)$ if $x^* < x \leq 3$. Also, $f'(x) > 0$ for x between 1 and x^*, so $f(x)$ is increasing in $[1, x^*]$. Therefore $f(x^*) > f(x)$ if $1 \leq x < x^*$.

b) Suppose that $f(3) = 5$. Use information from the graph and the tangent line approximation for $f(x)$ to find an approximate value of $f(3.04)$. Briefly explain using calculus and information from the graph why your approximate value for $f(3.04)$ is greater than or less than the exact value of $f(3.04)$. **Answer** Linear approximation gives $f(3.04) \approx f(3)+f'(3)(.04)$. The graph supplies $f'(3) = -2$ so $f(3.04) \approx 5+(-2)(.04) = 4.92$. The tangent line to $y = f'(x)$ at $x = 3$ has slope > 0 (the graph of $y = f'(x)$ is *increasing* near $x = 3$) so the derivative of $f'(x)$ is positive there: $f''(x) > 0$ near $x = 3$, and $y = f(x)$ is concave up near $x = 3$. The approximate value is *less than* the exact value since the tangent line will lie below the graph of $y = f(x)$.