Version 1

Math 151:4,5,6 **Additional points for your second exam grade** 11/30/2006

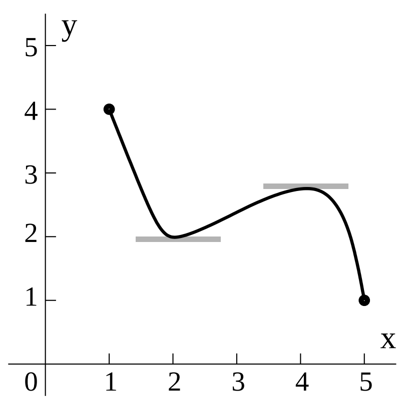
Some valid answers to the questions

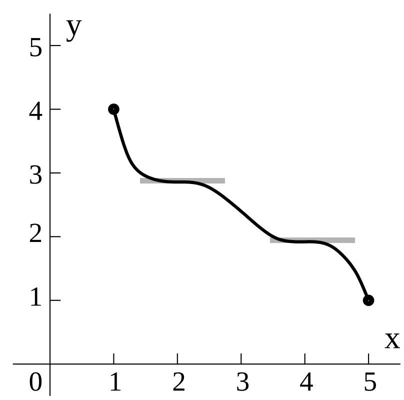
NO CALCULATORS OR NOTES ARE ALLOWED.

1. Suppose $f(x) = x(x-2)^3$.
a) (2 points) The derivative of f, $f'(x)$, is $\underline{(x-2)^3 + 3x(x-2)^2}$.
b) (2 points) The second derivative of f, $f''(x)$, is $\underline{3(x-2)^2 + 3(x-2)^2 + 6x(x-2)}$.
$3(x-2)^2 + 3(x-2)^2 + 6x(x-2) = (x-2)\left(3(x-2) + 3(x-2) + 6x\right) = (x-2)(12x-12)$
c) (2 points) The second derivative of f is 0 at $\underline{x=2}$ and $\underline{x=1}$.

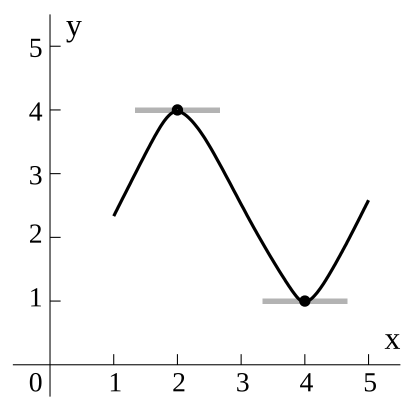
2. In this problem, f is a differentiable function and $f'(x) = 0$ only at $x = 2$ and $x = 4$.
a) (3 points) Use the axes to the right to draw a graph of f on the interval $[1,5]$ where the maximum value of f on that interval occurs at $x = 1$ and the minimum value of f on that interval occurs at $x = 5$. Also $f(1) = 4$ and $f(5) = 1$.

Here are two graphs which satisfy the requirements of this problem.





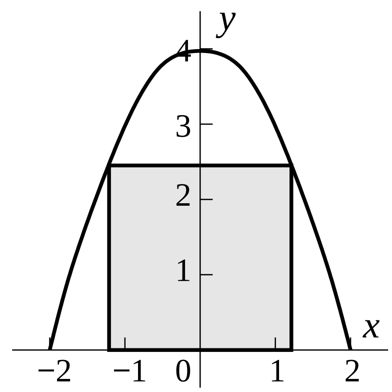
b) (3 points) Use the axes to the right to draw a graph of f on the interval $[1,5]$ where the maximum value of f on that interval occurs at $x = 2$ and the minimum value of f on that interval occurs at $x = 4$. Also $f(2) = 4$ and $f(4) = 1$.



3. (3 points) If $f(1) = 3$ and $f'(1) = 7$, then $f(.96)$ is approximately $\underline{3 + 7(-.04)}$.
(Do *not* simplify your numerical answer!)

OVER

4. (8 points) Find the area of the rectangle of largest area which has one side on the x-axis and is "under" the graph $y = 4 - x^2$, as shown to the right. (Do *not* simplify your numerical answer!)



Answer Call the area, A. Since A is the height times the width, we can write $A = 2x(4 - x^2)$ where $0 \leq x \leq 2$. Notice that A is 0 when $x = 0$ and when $x = 2$ so that the maximum will occur inside the interval, at a critical point. $A' = 2(4-x^2)+2x(-2x) = 8-6x^2$ and the only critical point inside the domain is at $x = \sqrt{\frac{8}{6}}$. There $A = 2\sqrt{\frac{8}{6}}\left(4 - \frac{8}{6}\right)$ which is positive, so this *is* the largest area.

The area of the rectangle of largest area is $\underline{2\sqrt{\frac{8}{6}}\left(4 - \frac{8}{6}\right)}$.

5. (7 points) The following is known about $f(x)$:

- $f''(x) = 5x + 3\cos x$
- $f'(0) = 7$
- $f(\pi) = -2$

Find $f(x)$. Do not attempt to "simplify" your answer, except that you must find explicit values of any trig functions.

Answer $f'(x) = \frac{5}{2}x^2 + 3\sin x + C$ so $f'(0) = C$ and since $f'(0) = 7$, this C is 7. Since $f'(x) = \frac{5}{2}x^2 + 3\sin x + 7$, $f(x) = \frac{5}{6}x^3 - 3\cos x + 7x + C$ and $f(\pi) = \frac{5}{6}\pi^3 - 3\cos\pi + 7\pi + C = \frac{5}{6}\pi^3 + 3 + 7\pi + C$ and this is supposed to be -2. Therefore $C = -2 - \frac{5}{6}\pi^3 - 3 - 7\pi$.

$f(x) = \underline{\frac{5}{6}x^3 - 3\cos x + 7x - 2 - \frac{5}{6}\pi^3 - 3 - 7\pi}$