#2 **2/2/97**

Some problems for 464

1. a) Show that $\dfrac{f(a+h)-2f(a)+f(a-h)}{h^2} = f''(a)+O(h^2)$.

b) The single most important partial differential operator in mathematical physics is the Laplacian, $\triangle = \dfrac{\partial^2}{(\partial x)^2} + \dfrac{\partial^2}{(\partial y)^2}$. Suppose $F(x,y)$ is a function of two variables. Find a "discrete" approximation to $\triangle F$ using part a) and give an O-error estimate.

Note The answer to b) is used in computer codes to approximate solutions of many physically interesting partial differential equations.

2. Consider the function

$$G(x) = \int_0^x \frac{1}{(1+t^2)(1+t)}\,dt$$

a) Find a power series expression for $G(x)$ centered at 0. Try to present the general pattern for the series.

Hint Partial fractions.

b) I have been advised that $G(.5) = .76805\ 59835 \pm 10^{-10}$. Find an overestimate for the number of terms of the power series which would be needed to get this result. Do this by finding some estimate for the "infinite tail" which would be thrown away.

Note You are *not* asked for a best possible estimate – just <u>some</u> estimate.

3. The asymptotic behavior of a function is frequently more interesting than specific values in applications. Here we ask you to get some asymptotic information about the values of the function f which appeared in problem 3 of the first assignment for very small and very large values of its argument. Recall that f is defined by

$$f(x) = \int_0^x \frac{1}{\sqrt{1+t^3}}\,dt$$

a) Show that when x is small, $f(x) = x - \frac{1}{8}x^4 + \frac{3}{56}x^7 + O(x^{10})$.

Hint Use the Binomial Theorem with exponent $-\frac{1}{2}$, plug in t^3, and integrate.

b) Show that when x is large, $f(x) = C - 2x^{-1/2} + \frac{1}{7}x^{-7/2} - \frac{3}{52}x^{-13/2} + O(x^{-19/2})$ where C is a constant which is approximately 2.80436.

Hint A silicon friend told me that $\int_0^\infty \frac{1}{\sqrt{1+t^3}}\,dt \approx 2.80436$. You may assume this and call the number, C. Then $f(x) = C - \int_x^\infty \frac{1}{\sqrt{1+t^3}}\,dt$. Analyze this integral using the substitution $t = \frac{1}{s}$ followed by some algebra and manipulations similar to that done in a).