Math 403, section 1

Entrance "exam"

Due at the beginning of class, Monday, January 21, 2002

1. (4) Compute $\displaystyle\int_1^2 \frac{dx}{1-2x}$ and simplify.

2. (8) If $u(x,y) = e^{y^2 x}$, what are $\dfrac{\partial^5 u}{\partial x^5}$ and $\dfrac{\partial^2 u}{\partial y^2}$?

3. (10) Find a function $v(x,y)$ such that $\dfrac{\partial v}{\partial x} = x + y$ and $\dfrac{\partial v}{\partial y} = x - y$.

4. (6) What shape (square, line, disk, circle, etc.) is described by the collection of ordered pairs $\{(2+\cos\theta, \sin\theta)\}$ when $0 \le \theta \le \pi$? Sketch this shape.

5. (8) Find all values of x for which the series $\displaystyle\sum_{n=0}^{\infty} (x-2)^n$ converges.

6. (4) The arctangent function, $\arctan x$, is an antiderivative of $\dfrac{1}{1+x^2}$. Based on this fact or otherwise, find the exact value of $\displaystyle\int_0^4 \frac{dy}{1+4y^2}$.

7. (10) Suppose λ is a positive real number. Define I_λ by

$$I_\lambda = \int_2^{14} e^{-\lambda x^3}\, dx\,.$$

a) Get a simple overestimate* for I_λ and apply this estimate to verify that $I_7 < \frac{1}{203}$.

b) Use the general estimate obtained in a) to show that $\lim_{\lambda\to\infty} I_\lambda = 0$.

8. (10) Compute the line integral of $x^2\, dx + xy\, dy$ along two paths: the straight line from $(0,0)$ to $(1,2)$, and the parabolic arc $y = 2x^2$ from $(0,0)$ to $(1,2)$.

Rules Please treat this as any other homework assignment. That is, you may consult textbooks or acquaintances or me (!), but the written work you hand in must be your own. An answer alone will not receive full credit – you must show supporting computation or give some explanation or both. I will grade what you hand in as an exam. A passing grade will be at least 75% of the 60 points. Familiarity with all of the material tested here is necessary for success in this course.

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* Draw a graph and then draw a bounding box.