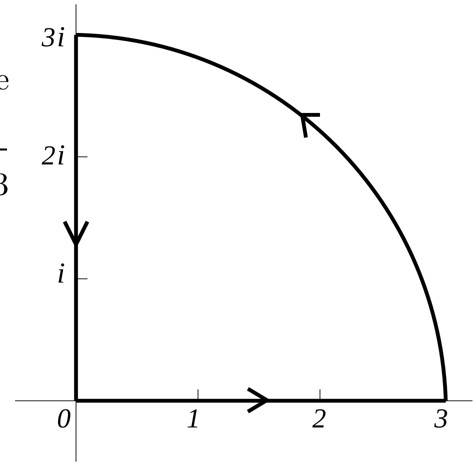
(12) 1. Let C be any simple closed curve described in the positive sense in the z plane, and write

$$g(w) = \int_C \frac{z^3 - 5z}{z - w}\, dz\,.$$

a) Show that $g(w) = 2\pi i(w^3 - 5w)$ if w is inside C.

b) Show that $g(w) = 0$ when w is outside C.

(12) 2. Compute $\displaystyle\int_B \frac{z^4}{(z-(1+i))^3}\,dz$ where B is the simple closed curve shown: the line segment from 0 to 3, followed by the quarter-circular arc centered at 0 from 3 to $3i$, followed by the line segment from $3i$ to 0.



Answer -24π

(12) 3. Suppose f is the function defined by

$$f(z) = \frac{z+1}{z(z-1)}.$$

Find the Laurent series representing $f(z)$ in the annulus $0 < |z| < 1$. Be sure to explain *why* the series you write is valid in that annulus. Find explicit values of the coefficients of z^{10} and z^{-10} in the series.

(Partial) **Answer** The coefficients are -2 and 0.

(14) 4. a) Suppose that f is an entire function and there is a positive constant K so that $|f(z)| > K$ for all z. Prove that f must be a constant function.

Hint what can you do with something that is *not* 0?

b) The exponential function is never 0 and is an entire function. Briefly explain why the exponential function does not contradict the assertion in part a).

(14) 5. If

$$F(z) = \frac{1}{z(z^2-1)(z^2+6)}$$

compute the integral of F over the circle of radius 2 centered at 0, oriented counterclockwise as usual. Note that $\sqrt{6} > 2$.

Answer $-\dfrac{\pi i}{21}$

(12) 6. Suppose the following is known about the coefficients of a power series $\sum_{n=0}^{\infty} a_n z^n$:

All a_n's are complex numbers with $|a_n| \leq 12$.

Explain why the power series converges for all z with $|z| \leq \frac{1}{10}$. Also verify that for those z's the sum of the series is always within a closed disc of radius 17 centered at 0.

Comment 17 is an overestimate!

(12) 7. The function $g(z) = (1+3z)e^{(z^2)}$ is analytic near 0.

a) Use results about power series of familiar functions to find terms up to and including degree 4 in the Taylor series of g centered at 0.

b) Use your answer to a) to compute $g^{(4)}(0)$.

Answer 12

(12) 8. a) Use results about power series of familiar functions to find an exact value of L:

$$\lim_{z\to 0} \frac{(\cos z) - 1 + \dfrac{z^2}{2!}}{z^4} = L.$$

b) Suppose the function h is defined by

$$h(z) = \begin{cases} \dfrac{(\cos z) - 1 + \dfrac{z^2}{2!}}{z^4} & \text{if } z \neq 0 \\ L & \text{if } z = 0 \end{cases}$$

where L is the number found in a). Explain carefully why h is entire (analytic in all of the complex numbers).

Second Exam for Math 403, section 5

April 16, 2001

NAME ______________________________

Do all problems, in any order.
Show your work. An answer alone will not receive full credit.
Answers must be supported by computation and explanation.
No notes or calculators may be used on this exam.

Problem Number	Possible Points	Points Earned:
1	12	
2	12	
3	12	
4	14	
5	14	
6	12	
7	12	
8	12	
Total Points Earned:		