640:291:01 Part III: playing with calculus on Maple 8/28/2002

The basic calculus commands do differentiation and integration. Let's try them immediately.

diff(3*x^7 - 22.1*x^2,x); RET

 and

int(x*sqrt(x+2),x); RET

Please realize that this integration is not too easy: Maple either substituted or it integrated by parts (it can do both).

Probably Maple knows all the functions you do, and a number of others. The function $\sin(x^2\sqrt{x+1})$ and the appropriate calculus rules are known. Let's first assign this expression a name and then play.

Q:=sin(x² * sqrt(x+1)); RET

diff(Q,x); RET

This should get the first derivative. How many ways can you think of to get the second derivative? Here are a few. First, immediately after typing the command above and getting a response, type

diff(% ,x); RET

This will do it. For an independent computation, try the command line

diff(Q,x,x); RET

How about the tenth derivative? First type

x\$10; RET

to see what the symbol \$ does. Please now find the 10^{th} derivative of the function $\sin(x^2\sqrt{x+1})$ with very little typing. You may now realize (even if you want to compute the 10^{th} derivative of this function) why people end commands with : (which turns off the output) rather than with ; which displays the output. You may want the results for some purpose, but you may not have the need or desire to actually look at it!

What is the coefficient of x^3 in the seventh derivative of $\left(x^2 + \frac{1}{x^2}\right)^5$? First compute the indicated derivative.

You'll get a mess. Then have Maple massage the result algebraically so you can read off the answer. I'm an amateur and first tried expand (%) and I also tried simplify (%) and the displayed results were different

(but equivalent algebraically). You can also try it by first asking Maple to expand $\left(x^2 + \frac{1}{x^2}\right)^5$ and then differentiate the result seven times. I hope the answer will be the same.

Let's look at integration a bit more closely. Define V to be $e^{\sin x}$:

V:=exp(sin(x)); RET

Now let's integrate it. First (read and try this carefully!) type

int(V,w); RET

and explain the result to yourself. Remember, a program will do what you tell it to do! Now try int(V,x); RET

and you may need to wait a bit and then have something else to explain. Maple knows the usual integration algorithms and many additional antidifferentiation tricks. An answer like this is a fairly good hint that it "can't be done": that is, the antiderivative can't be expressed in terms of familiar functions with familiar ways of combining them, including sum, product, composition, ... even using the rather large collection of functions Maple knows.

We can also compute definite integrals. For example,

computes $\int_{\frac{1}{7}}^{v} x^{3} dx$ (if you want it!). Maple indicates a range (for integration and for other purposes) by the notation variable=lower limit.upper limit.

Remember V? Let's be sure Maple does (check by typing V;) and then compute $\int_0^1 e^{\sin x} dx$ by typing

int(V,x=0..1); RET

and consider the result. Perhaps we are disappointed, but I can be happier if I follow that answer with evalf(%); RET

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You can evaluate V itself with a command like

 $subs({x=3},V); RET$

followed by

If you're not scared, you could have done this all together by typing

evalf(subs({x=3},V)); RET

but sometimes I get confused by the matching required (in count and type) of all the parentheses. We could similarly evaluate a derivative of V by differentiating with diff, substituting, and then evalfuating. Or we could define our own functions. Initially the syntax may seem burdensome, but let's try it with a simple example.

 $N:=x-->\arctan(x^3); RET$

This means: let's call N the function which assigns $\arctan(x^3)$ to the input x. The "arrow" is gotten by typing a dash followed by the "less than" symbol. Type

N(2); RET

 and

N(2*z); RET

to make sure that Maple understands that N is a function. Now type

diff(N(x), x); RET

That's now an **expression** and not a function. There's no way to "plug in" another expression like K + 3 in it easily (yes, we could use the subs command, but that's cumbersome). The designers of Maple have another way to differentiate functions (such as N) rather than expressions (such as Q). Try

D(N); RET

and view the result. Indeed: call it by a new name, say, M:

M:=D(N); RET

Now evaluate M(3) and M(K+3).

Please check out the difference between

int(N(x),x); RET

 and

int(N,x); RET

I find one of these answers amusing to look at (because I didn't have to compute it and I can appreciate the work involved!) and the other seems almost silly: the function hasn't been told what to evaluate, so Maple can't integrate.

The difference between "expressions" like V and "functions" like N can be subtle. Expressions seem more static, while functions have a more dynamic aspect – substituting is built into their structure.

Maple also knows about such calculus topics as limits, sums, and series. I suggest that when you need to work with any of these, at that time please try help(limit) and help(sum) and help(series). I almost always look at the examples first. Don't be afraid to try some experiments, and look at the SEE ALSO pages if the command you are investigating doesn't do exactly what you want.

Please see if you can answer this problem using only Maple without writing any intermediate results!

Suppose A is a positive constant. Compute $\int_0^{\infty} (\sin Ax) e^{-x} dx$. For which value of A is the

integral largest? What is the value of that largest integral? It's time to graph.

Disclaimer! Non-advertisement!! Important information!!!

Symbolic manipulation programs such as Maple are increasingly available. Mathematica and Derive are other programs with the same capability. There are many special purpose programs in science, engineering, and mathematics which have extensive "intelligence" to analyze models. We're considering Maple here because Rutgers has a site license for this program. It is generally available on Rutgers systems. The specific instructions won't be the same from program to program, but many of the same ideas will be present. Students should expect to have a machine do tiresome or elaborate symbolic computations as well as numerical computations.