x	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	π	$3\pi/2$	2π
$\sin x$	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1	0	-1	0

x	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	π	$3\pi/2$	2π
$\cos x$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0	-1	0	1

$\sin(2x) = 2\sin x\cos x$

$\cos(2x) = \cos^2 x - \sin^2 x$

$f(x)$	$\int f(x)\,dx$
$x^r, r \neq -1$	$x^{r+1}/(r+1) + C$
x^{-1}	$\ln\lvert x\rvert + C$
e^x	$e^x + C$
$a^x, a \neq 1$	$a^x/(\ln a) + C$

$f(x)$	$\int f(x)\,dx$
$\sin x$	$-\cos x + C$
$\cos x$	$\sin x + C$
$\sec^2 x$	$\tan x + C$
$\sec x\tan x$	$\sec x + C$

$f(x)$	$\int f(x)\,dx$
$1/\sqrt{a^2-x^2}, a \neq 0$	$\arcsin(x/a) + C$
$1/(a^2+x^2), a \neq 0$	$(1/a)\arctan(x/a) + C$
$\tan x$	$-\ln\lvert\cos x\rvert + C$
$\sec x$	$\ln\lvert\sec x + \tan x\rvert + C$

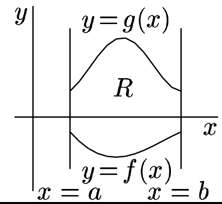
Areas and Volumes of Solids of Revolution, see figure at the right

R is the region bounded by $y = g(x)$, $y = f(x)$, $x = a$, $x = b$, with $g(x) \geq f(x)$ for x in $[a, b]$.

Area of R: $\int_a^b (g(x) - f(x))\,dx$

Volume of solid found by rotating R around the x-axis: $\int_a^b \pi((g(x))^2 - (f(x))^2)\,dx$, $(f(x) \geq 0)$

Volume of solid found by rotating R around the y-axis: $\int_a^b 2\pi\, x(g(x) - f(x))\,dx$, $(a \geq 0)$



Integration by parts: $\int u\,dv = uv - \int v\,du$. Choose dv to be easy to integrate and so that $\int v\,du$ is simpler than $\int u\,dv$.
Use for integrals of: $x^m e^{ax}$, $x^m \sin ax$, $x^m \cos ax$, $x^m \ln x$, $x^m \arctan x$, $x^m \arcsin x$, $e^{ax}\sin bx$, $e^{ax}\cos bx$, $\sec^m x$, m odd.

$\int \sin^m x\cos^n x\,dx$: Reduce to a sum of integrals of the type: $\int \sin^j x(\cos x\,dx)$ and $\int \cos^k x(\sin x\,dx)$
m odd: group $\sin x$ with dx. Replace $\sin^{m-1}x$ using $\sin^2 x = 1 - \cos^2 x$. Let $u = \cos x$, $du = -\sin x\,dx$. Expand.
n odd: group $\cos x$ with dx. Replace $\cos^{n-1}x$ using $\cos^2 x = 1 - \sin^2 x$. Let $u = \sin x$, $du = \cos x\,dx$. Expand.
m, n both even: Use $\sin^2 x = (1 - \cos(2x))/2$, $\cos^2 x = (1 + \cos(2x))/2$. Possibly repeat, or use an earlier case.
EX: $\int \sin^5 x\cos^4 x\,dx = \int \sin^4 x\cos^4 x(\sin x\,dx) = \int (1-\cos^2 x)^2\cos^4 x(\sin x\,dx) = \int (1-u^2)^2 u^4(-du) = \int(-u^4 + 2u^6 - u^8)du = -(1/5)\cos^5 x + (2/7)\cos^7 x - (1/9)\cos^9 x + C$. EX: $\int \sin^2 x\cos^2 x\,dx = \int (1/4)(1-\cos(2x))(1+\cos(2x))dx = (1/4)\int(1-\cos^2(2x))dx = (1/4)\int(1-(1/2)(1+\cos(4x)))dx = (1/8)\int(1-\cos(4x))dx = (1/8)x - (1/32)\sin(4x) + C$.
$\int \sec^m x\tan^n x\,dx$: Reduce to a sum of integrals of the type: $\int \sec^j x(\sec x\tan x\,dx)$ and $\int \tan^k x(\sec^2 x\,dx)$
m even: group $\sec^2 x$ with dx. Replace $\sec^{m-2}x$ using $\sec^2 x = 1 + \tan^2 x$. Let $u = \tan x$, $du = \sec^2 x\,dx$. Expand.
n odd: group $\sec x\tan x$ with dx. Replace $\tan^{n-1}x$ using $\tan^2 x = \sec^2 x - 1$. Let $u = \sec x$, $du = \sec x\tan x\,dx$.
m odd, n even: Use $\tan^2 x = \sec^2 x - 1$ to express as a sum of integrals of $\sec^m x$, m odd. Use integration by parts.

Integrals involving $\sqrt{a^2 - x^2}$: set $x = a\sin\theta$. Then $dx = a\cos\theta\,d\theta$ and $\sqrt{a^2-x^2} = a\cos\theta$.
Integrals involving $\sqrt{a^2 + x^2}$: set $x = a\tan\theta$. Then $dx = a\sec^2\theta\,d\theta$ and $\sqrt{a^2+x^2} = a\sec\theta$.
EX: $\int x^5(1-x^2)^{3/2}\,dx = \int \sin^5\theta\cos^3\theta(\cos\theta\,d\theta) = \int \sin^5\theta\cos^4\theta\,d\theta = -(1/5)\cos^5\theta + (2/7)\cos^7\theta - (1/9)\cos^9\theta + C = -(1/5)(1-x^2)^{5/2} + (2/7)(1-x^2)^{7/2} - (1/9)(1-x^2)^{9/2} + C$

Integrals of rational functions: $\int (f(x)/g(x))\,dx$, with $f(x), g(x)$ polynomials.
(1) If $\deg f(x) \geq \deg g(x)$, divide $g(x)$ into $f(x)$ using long division of polynomials to get quotient $q(x)$ and remainder $r(x)$ with $\deg r(x) < \deg g(x)$. Then $\int (f(x)/g(x))\,dx = \int q(x)\,dx + \int (r(x)/g(x))\,dx$. Use next method on last integral.
(2) If $\deg f(x) < \deg g(x)$, use the method of partial fractions. First factor $g(x)$ into a product of linear factors $x + A$ and quadratic factors $x^2 + Bx + C$ with no real roots, or $g(x) = \underbrace{(x+A)^m\cdots}_{\text{linear factors}}\underbrace{(x^2+Bx+C)^m\cdots}_{\text{quadratic factors}}$.

Then, $f(x)/g(x)$ is a sum of terms:

$$\underbrace{\frac{D_1}{(x+A)} + \frac{D_2}{(x+A)^2} + \cdots + \frac{D_m}{(x+A)^m}}_{m \text{ terms for each linear factor } (x+A)^m} \text{ and } \underbrace{\frac{E_1x+F_1}{(x^2+Bx+C)} + \frac{E_2x+F_2}{(x^2+Bx+C)^2} + \cdots + \frac{E_nx+F_n}{(x^2+Bx+C)^n}}_{n \text{ terms for each quadratic factor } (x^2+Bx+C)^n}$$

To find the constants D_i, E_j, F_k, multiply both sides by $g(x)$. Then multiply the terms out and equate the coefficients of the same powers of x on each side of the equation. Finally, solve the resulting system of linear equations.
EX: $\frac{x^2+x+1}{x^2(x-1)(x^2+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1} + \frac{Dx+E}{x^2+1}$. First multiply both sides by $x^2(x-1)(x^2+1)$ to obtain: $x^2 + x + 1 = Ax(x-1)(x^2+1) + B(x-1)(x^2+1) + Cx^2(x^2+1) + Dx^3(x-1) + Ex^2(x-1)$, or $x^2 + x + 1 = (A+C+D)x^4 + (-A+B-D+E)x^3 + (A-B+C-E)x^2 + (-A+B)x + (-B)$. Thus, $A+C+D = 0$, $-A+B-D+E = 0$, $A-B+C-E = 1$, $-A+B = 1$, $-B = 1$. Solving, $A = -2$, $B = -1$, $C = 3/2$, $D = 1/2$, $E = -1/2$.
If $g(x)$ is a product of distinct linear factors, there is a short cut. Multiply both sides by $g(x)$ as before.
EX: $\frac{x^2-2}{(x-2)x(x+1)} = \frac{A}{x-2} + \frac{B}{x} + \frac{C}{x+1}$. Then: $x^2 - 2 = Ax(x+1) + B(x-2)(x+1) + C(x-2)x$. Let x equal each root of $g(x)$: $x = 2, 0, -1$. Then $x = 2$ gives $2 = 6A$; $x = 0$, $-2 = -2B$; $x = -1$, $-1 = 3C$, so $A = 1/3$, $B = 1$, $C = -1/3$.

Suppose the integral $\int_a^b f(x)\,dx$ is approximated by dividing $[a, b]$ into n equal segments. If M_j is an overestimate for $|f^{(j)}(x)|$ on $[a, b]$, the trapezoidal error is at most $M_2(b-a)^3/12n^2$ and the Simpson's error is at most $M_4(b-a)^5/180n^4$.

Definitions: $\int_a^\infty f(x)\,dx = \lim_{t\to\infty}\int_a^t f(x)\,dx$; $\int_{-\infty}^b f(x)\,dx = \lim_{t\to-\infty}\int_t^b f(x)\,dx$. $\int_1^\infty \frac{dx}{x^a}$ converges if $a > 1$, diverges if $a \leq 1$.
Comparison: if $0 \leq f(x) \leq g(x)$, $I_f = \int_a^\infty f(x)\,dx$, $I_g = \int_a^\infty g(x)\,dx$ then: if I_g converges, so does I_f; if I_f diverges, so does I_g. Similar definitions and results for $\int_a^b f(x)\,dx$ when f has bad behavior ($\to +\infty$ or $-\infty$, say) at an endpoint. For example, $\int_0^1 \frac{dx}{x^a}$ converges if $a < 1$, diverges if $a \geq 1$. Some limits: if $a > 0$, $\lim_{x\to\infty} x^a = \infty$, $\lim_{x\to\infty} 1/x^a = 0$. If $a > 1$, $\lim_{x\to\infty} a^x = \infty$, $\lim_{x\to\infty} 1/a^x = 0$. Logs: $\lim_{x\to\infty}\ln x = \infty$; $\lim_{x\to 0^+}\ln x = -\infty$. And $\lim_{x\to\pm\infty}\arctan x = \pm\pi/2$.
L'Hospital: if $\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{0}{0}$ or $\frac{\pm\infty}{\pm\infty}$, then $\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$ (when the latter limit exists).