(9) 1. The equation

$$(4 - x)y^2 = x^3$$

defines a curve. Write an equation for a line tangent to this curve at the point $(2, 2)$.

(10) 2. Use calculus to locate the maximum and minimum values of the function $f(x) = x^3 - 12x$ on the closed interval $[0, 4]$.

(21) 3. Suppose that $F(x) = e^x(2 - x)$.

a) Where is $F(x) = 0$?

b) Compute $F'(x)$. Where is $F'(x) = 0$?

c) Compute $F''(x)$. Where is $F''(x) = 0$?

THIS PROBLEM CONTINUES ON THE NEXT PAGE. →

d) Sketch a graph of $y = F(x)$. Use calculus to identify the following features on your graph. Describe each of these algebraically and label each one clearly on your graph.

- Intervals where F is increasing and where it is decreasing.

  Answer ____________________

- Intervals where F is concave up and where it is concave down.

  Answer ____________________

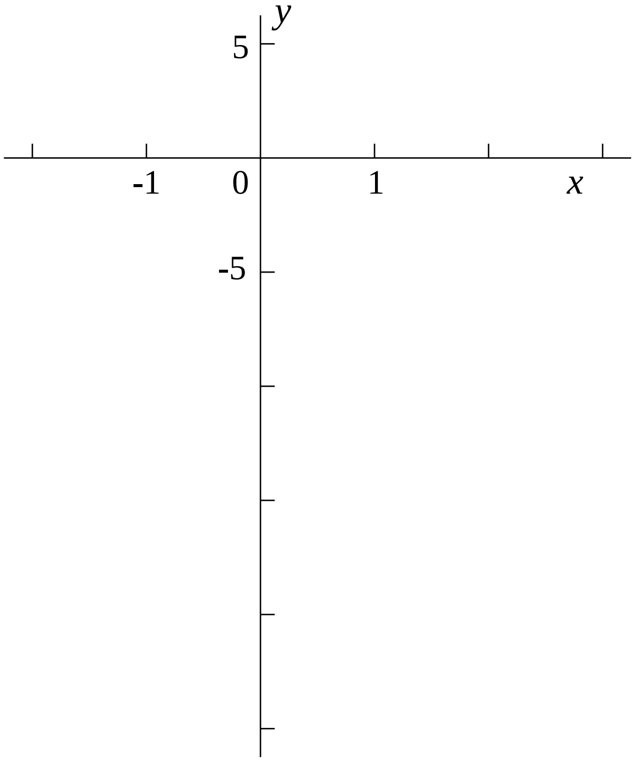
- The coordinates of any maxima or minima of F.

  Answer ____________________

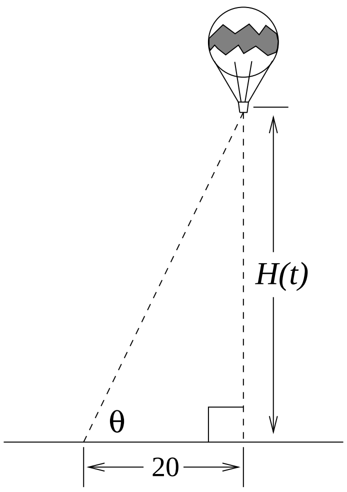
- The coordinates of any inflection points of F.

  Answer ____________________

Note the difference in scale on the vertical and horizontal axes drawn below. Remember to *label* your graph!



(10) 4. An observer is standing 20 feet away across level ground from the site of a balloon launch. The balloon travels straight up. Suppose that at time t, the bottom of the balloon's gondola is $H(t)$ feet from the ground.

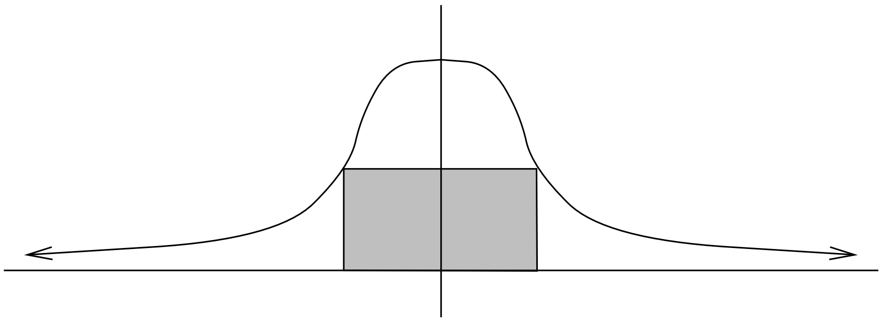


a) Write an equation relating $H(t)$ and θ, the angle seen by the observer between the ground and the bottom of the gondola.

You will need to use a trigonometric function.

b) At a certain time, the bottom of the gondola is 50 feet off the ground and is rising at 6 feet per second. At that time, what is the angle θ defined in part a) and what is the rate of change of that angle?

(14) 5. Find the dimensions of the rectangle with *largest* area which has one side on the x-axis and two corners on the curve $y = \dfrac{1}{3+x^2}$. Explain briefly why you have found the largest such rectangle.

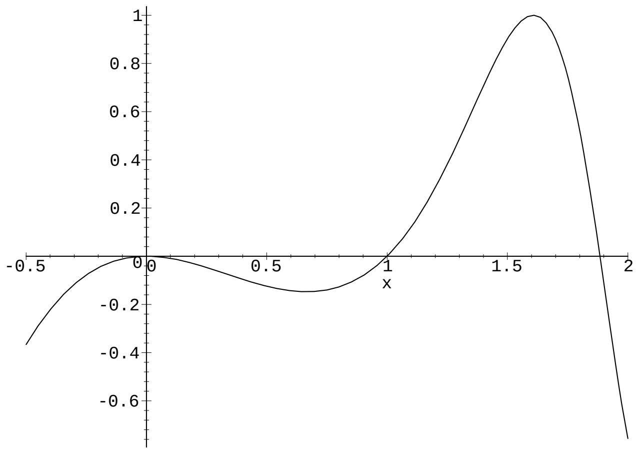


(12) 6. Suppose $g(x) = \sqrt{1+3x^2}$. Then $g(1) = 2$.

a) Use the differential or tangent line approximation to find an approximate value for $g(1.04)$.

b) The true value of $g(1.04)$ is $2.06029\,12415$ to ten-decimal place accuracy. Use calculus to explain why the approximation you found in a) is greater than or less than the true value.

(24) 7. The function $H(x)$ has domain $-.5 \leq x \leq 2$ and $H'(x) = \sin(x^3 - x^2)$. A graph of <u>the derivative of H</u> is shown below. Use the graph of H' to answer the questions below. Don't compute H'' algebraically, and don't try to get a formula for H.



The graph of $H'(x)$, the **derivative** of $H(x)$.

a) Does H have any critical numbers? Write the approximate value of each critical number.

b) Which of the critical numbers are relative maxima, which are relative minima, and which are neither? Give brief reasons for each of your answers.

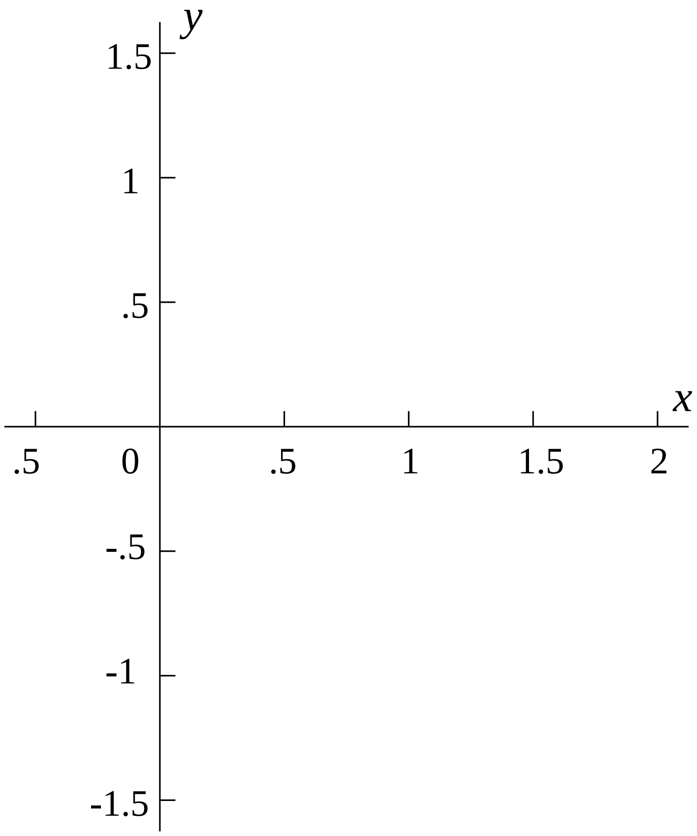
c) In which intervals is H concave up? In which intervals is H concave down?

d) Does H have any inflection points? Write the approximate first (x) coordinate of each inflection point.

THIS PROBLEM CONTINUES ON THE NEXT PAGE. →

e) Sketch a possible graph of $y = H(x)$ using all the information you have obtained in previous parts of this problem.

Label each relative minimum with an **m** and each relative maximum with an **M** and each inflection point with an **I**.



B B

Exam 2 for Math 135

Sections 1, 2 and 3

April 10, 1998

NAME (*please print*): ____________________

SIGNATURE: ____________________

SECTION #: ____________________

Do all problems, in any order.

Show all your work. Full credit may not be given for an answer alone.

You may use one sheet of notes and any standard calculator without a QWERTY keypad on this exam. You may use no other materials.

Problem Number	Possible Points	Points Earned:
1	9	
2	10	
3	21	
4	10	
5	14	
6	12	
7	24	
Total Points Earned:		

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