4/15/98

Review ADVICE for the final exam in Math 135
(including some review problems)

These review problems mostly cover the last few weeks of the course. The final is cumulative and will cover all parts of the course. Material which has not yet been tested will be emphasized more on the final exam. Keep in mind that if we thought it was important earlier, we probably still think it is important. Here are other useful references.

- The earlier review sheets (with answers available on the web).
- The exams *you* had earlier in this course.
- Two real exams with answers given this semester to one class (with answers and grading guide available on the web).
- **Three real final exams** given during the last two academic years (available on the web).

Math 135 material on the web including answers to review questions can be accessed through:

`http://www.math.rutgers.edu/math135.html`

The final exam will be given **Wednesday, May 5, from 4 to 7 PM** with locations to be announced separately for each class. These review problems are intended to represent the type of problems which *may* appear on exams. Many routine review problems are available in the review sections of the text.

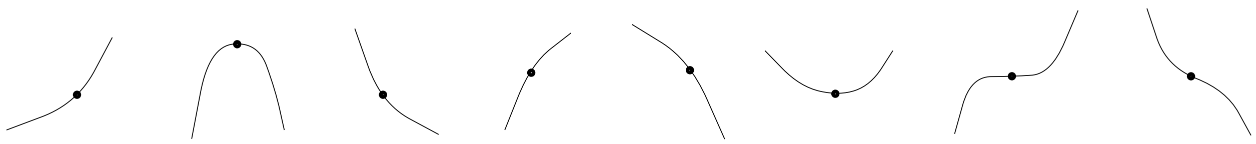
Graphing calculators may be used on the Math 135 final exam but calculators and computers with QWERTY keyboards or symbolic differentiation and integration programs are not allowed. Students may bring one sheet of paper containing notes (using both sides!) up to 8.5 by 11 inches in size to any exam.

0. Suppose $f(x) = Ce^{Dx}$. Find values of C and D so that $f(1) = 3$ and $f(2) = 4$. Sketch a graph of $y = f(x)$ for x between -2 and 6.

x	$Q(x)$	$Q'(x)$	$Q''(x)$
0	1	-4	-3
1	2	0	7
2	0	-5	2
3	1	0	-2
4	5	5	-2
5	8	-1	0
6	-2	2	3
7	-2	0	0

1. Suppose Q and its first derivative Q' and its second derivative Q'' have the values indicated in the accompanying table.

Below are some disconnected pieces of the graph of Q. Each value of x matches exactly one picture. Find the matches.



A B C D E F G H

2. Graph $y = (2x+1)e^{-x^2}$. Use calculus to give information as precisely as possible about local extrema, intervals of increase and decrease, inflection points, and intervals of concavity.

3. The line $y = 3x + 7$ is tangent to the graph of $y = f(x)$ when $x = 4$. What is $f(4)$? what is $f'(4)$?

4. Suppose y is defined implicitly as a function of x by $x^2 + Axy^2 + By^3 = 1$ where A and B are constants to be determined. Given that this curve passes through the point $(3, 2)$ and that its tangent at this point has slope -1, find A and B.

5. Evaluate these indefinite integrals.

a) $\int 7x^2 - 3e^x + \frac{5}{x}\,dx$ b) $\int (x^3+5)^2\,dx$ c) $\int 5\sin x + \cos(5x)\,dx$ d) $\int \frac{x}{x^2+5}\,dx$

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6. Find a solution to the differential equation $y' = 2x^3 - 1$ passing through the point $(2, 3)$.

7. What is the maximum value of the function $K(x)$ if $K''(x) = -9$ for all x and if also $K(0) = 0$ and $K(2) = 0$?

8. $\displaystyle\int_0^1 \frac{4}{1+x^2}\,dx$ and $\displaystyle\int_0^{\sqrt{3}} \frac{3}{1+x^2}\,dx$ are both exactly equal to π*. Use this together with simple properties of definite integrals to get the exact value of $\displaystyle\int_1^{\sqrt{3}} \frac{1}{1+x^2}\,dx$.

9. Evaluate these definite integrals using methods of calculus.

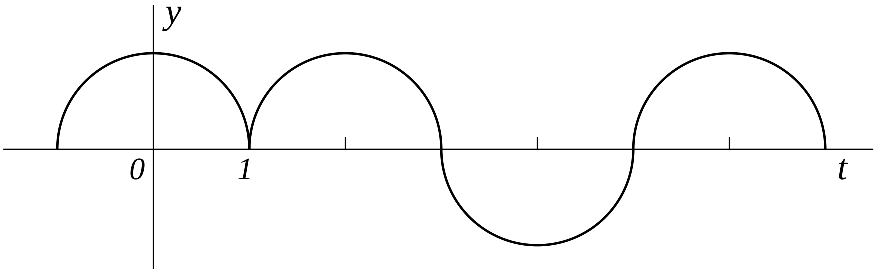
a) $\displaystyle\int_1^2 \left(x^3 - \frac{1}{x^4}\right) dx$ b) $\displaystyle\int_0^{\ln 3} 4e^{2x}\,dx$ c) $\displaystyle\int_0^2 x^2\sqrt{1+3x^3}\,dx$ d) $\displaystyle\int_{\ln\pi}^{\ln 2\pi} e^x \sin(e^x)\,dx$ e) $\displaystyle\int_e^{e^e} \frac{1}{x\ln x}\,dx$

10. a) It's known that $\displaystyle\int x^2e^x\,dx = Ex^2e^x + Fxe^x + Ge^x$ (+ any constant) for certain specific numbers E and F and G. What are these specific numbers E and F and G?

b) Use your answer to a) to compute $\displaystyle\int_0^1 x^2e^x\,dx$.

11. If $F(x) = \displaystyle\int_{-42}^{x} \frac{\sin(t^2)}{1+t^4}\,dt$, compute $F(-42)$, $F'(0)$, and $F'(\sqrt{\pi})$ exactly.

12. Below is a graph of $y = f(t)$.



Graph of $y = f(t)$

Each segment of the graph is a semicircle. Suppose that $g(x) = \displaystyle\int_{-1}^{x} f(t)\,dt$.

a) What are $g(-1)$ and $g(0)$ and $g(5)$? What are $g'(-1)$ and $g'(0)$ and $g'(5)$?

b) Compute a Riemann sum estimate for $g(3) = \displaystyle\int_{-1}^{3} f(t)\,dt$ associated with the partition $-1 < 0 < 1 < 3$ where the sample points are the left endpoints of the subintervals.

13. What is the area enclosed by $y = x^5$, the x−axis, and the lines $x = 1$ and $x = 2$?

14. First, find the area enclosed by $y = \sin x$, the x-axis, and the line $y = \dfrac{\pi}{2}$. Then find a vertical line which splits this area into two equal parts.

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* Really! This is true!