(10) 1. Evaluate the indicated limits exactly. Give brief evidence supporting your answers which is not based on a calculator graph or calculator computations.

a) $\lim_{x\to\infty} \dfrac{2x^2-5}{3x^2+1}$.

b) $\lim_{x\to 4} \dfrac{x-4}{\sqrt{x}-2}$.

(12) 2. Find the equations of all vertical and horizontal asymptotes of the function

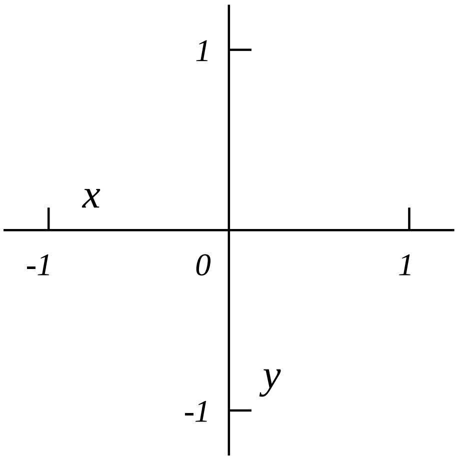
$$f(x) = \frac{3e^x+5}{7e^x-2}.$$

Computations with exp and log should be simplified as much as possible. A numerical approximation like 1.40135 is **not** acceptable.

(18) 3. a) Write the definition of derivative as a limit and *use this definition* to find the derivative of $F(x) = x - x^2$.

b) Use your answer to a) to find the equation of a line tangent to $y = x - x^2$ at the point where $x = 0$.

c) Sketch $y = x - x^2$ and the line found in b) on the axes given.



(14) 4. Find $\dfrac{dy}{dx}$ for each of the following:

a) $y = \dfrac{2x^2+5}{5x^3+1}$ b) $y = (4x+3)\sqrt{x^3+7}$ c) $xy^3 = \cos(7x+5y)$

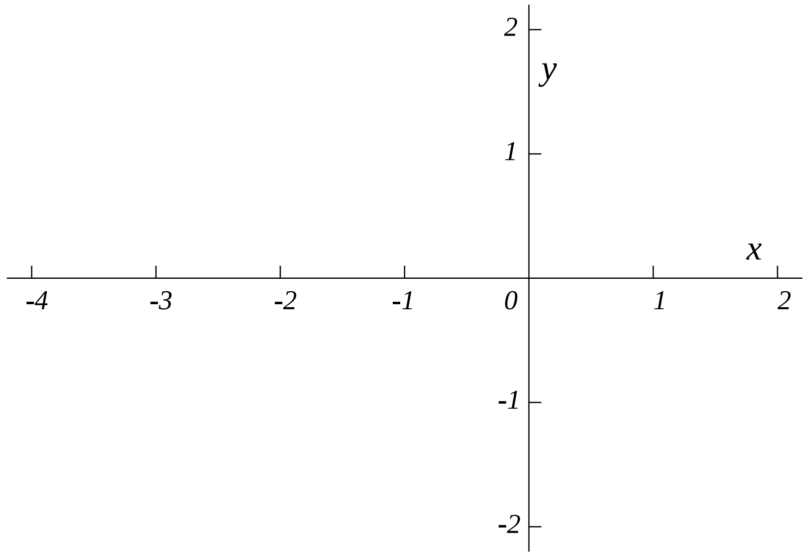
(16) 5. In this problem

$$W(x) = \begin{cases} x+3 & \text{if } x \le -2 \\ \frac{1}{2}x^2 + A & \text{if } -2 < x \end{cases}$$

where A is a constant to be determined in part a).

a) Find A so that the function is continuous for all values of x.

b) Sketch a graph of $y = W(x)$ for $-4 \le x \le 2$ using the value of A found in a) on the axes given.



c) Is $W(x)$ differentiable at $x = -2$ using the value of A you have found in part a)? Explain your answer briefly.

(18) 6. Suppose that $N(x) = 5x^3 - 3x^5$.

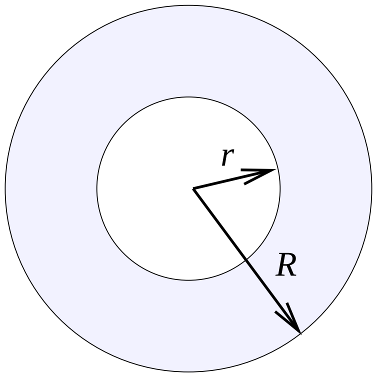
a) Compute $N'(x)$ and $N''(x)$. Where are each of these functions equal to 0?

b) Find all relative maximum and minimum values of $N(x)$. Briefly explain your answers using calculus.

c) Find all points of inflection of $N(x)$. Briefly explain your answers using calculus.

(12) 7. Two circles have the same center. The inner circle has radius r which is increasing at the rate of 3 inches per second. The outer circle has radius R which is increasing at the rate of 2 inches per second. Suppose A is the area of the region *between* the circles.

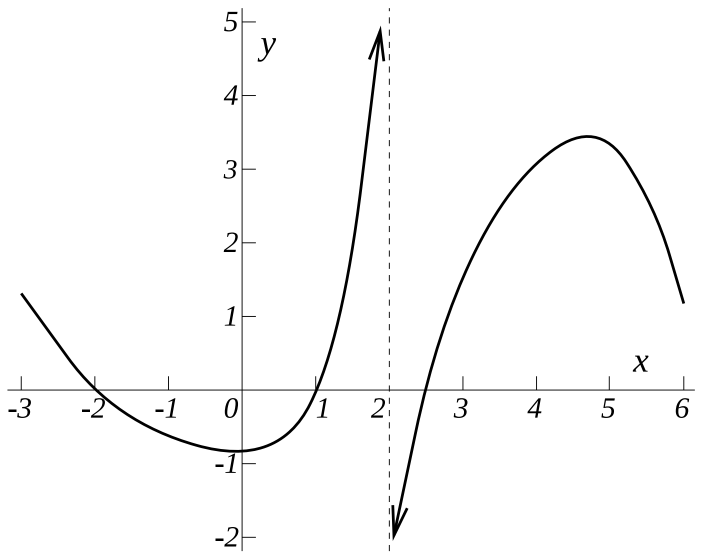
At a certain time, r is 7 inches and R is 10 inches. What is A at that time? How fast is A changing at that time? Is A increasing or decreasing at that time?



(16) 8. A box with an open top is to be made from rectangular sheet of cardboard 5 inches by 8 inches by cutting equal squares out of the four corners and bending up the resulting four flaps to make the sides of the box. Use calculus to find the largest volume of the box.

Be sure to explain briefly why your answer gives a maximum.

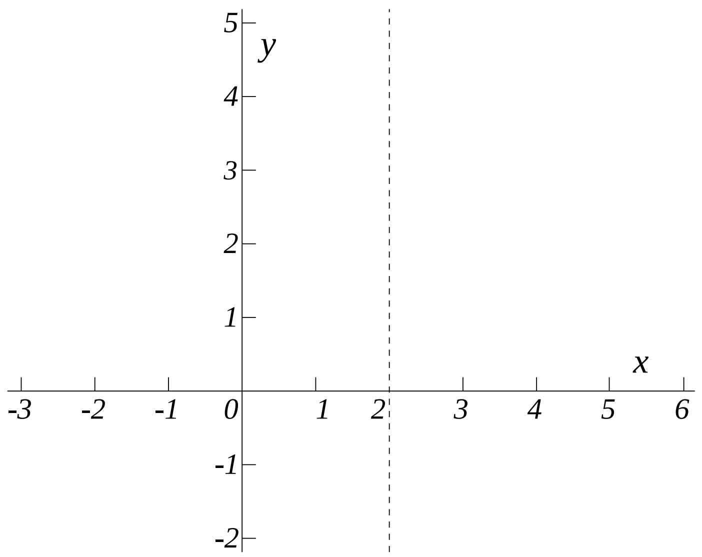
(18) 9. $h(x)$ is a function with domain $-3 \le x < 2$ and $2 < x \le 6$. Below is a graph of $h'(x)$, the derivative of $h(x)$.



The graph of $y = h'(x)$, the derivative of $h(x)$

Sketch a possible graph of $h(x)$ on the axes given below using information from the graph of $h'(x)$.

- Label any relative maxima of $h(x)$ with an **M** on your graph.
- Label any relative minima of $h(x)$ with an **m** on your graph.
- Label any points of inflection of $h(x)$ with an **I** on your graph.



- In what interval(s) on this graph is $h(x)$
  - increasing? **ANSWER** __________
  - decreasing? **ANSWER** __________
  - concave up? **ANSWER** __________
  - concave down? **ANSWER** __________

(8) 10. In this problem, $Q(x) = (5x^3 + 9)^{14} - (4 - 2x^5)^{13}$.

a) Compute $Q'(x)$.

b) Use your answer to a) and calculus to explain why $Q(200)$ is less than $Q(500)$.

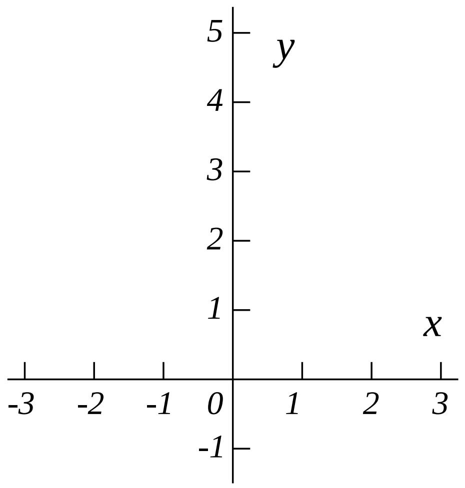
(8) 11. Suppose $k''(x) = x + \dfrac{1}{x^2}$ and $k(1) = 1$ and $k'(1) = -2$. Find a formula for $k(x)$. Computations with exp and log should be simplified as much as possible. A numerical approximation like 1.40135 is **not** acceptable.

(12) 12. a) Suppose P and Q are constants, and that $G(x) = P\sin(7x) + Qx\cos(7x)$. Compute $G'(x)$.

b) Find specific values of P and Q so that $G'(x) = x\sin(7x)$.

c) Use your answer to b) to evaluate $\displaystyle\int_0^{\pi/7} x\sin(7x)\,dx$ exactly.

(12) 13. a) Sketch the region in the plane bounded above by $y = 4 - x^2$ and below by the x-axis.



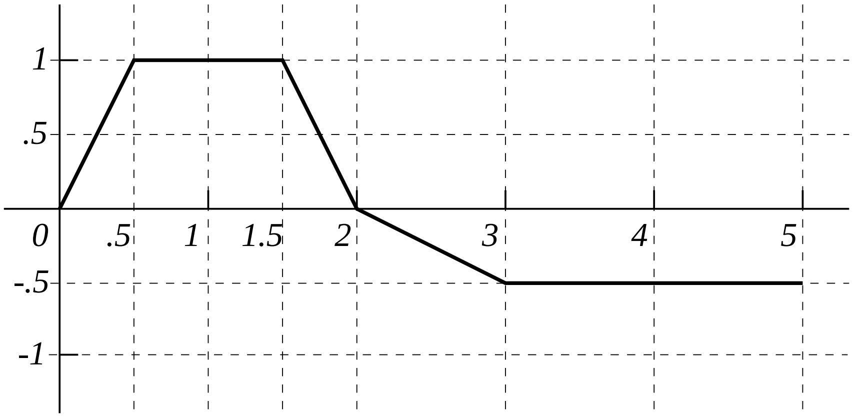
b) Find the area of this region using methods of calculus.

(16) 14. Evaluate the indicated integrals using methods of calculus.

Computations with exp and log and square root should be simplified as much as possible. A numerical approximation like 1.40135 is **not** acceptable.

a) $\displaystyle\int_1^4 \left(3x - 5\sqrt{x}\right)dx$ b) $\displaystyle\int_0^{\sqrt{\ln 7}} x e^{(x^2)}\,dx$

(10) 15. Below is a graph of the function R which is defined on the interval $[0,5]$. The graph is made up of straight line segments.



The graph of R

a) Compute the Riemann sum estimate for $\int_0^5 R(t)\,dt$ associated with the partition $0 < 1 < 2 < 4 < 5$ where the sample points are the midpoints of the subintervals.

b) Compute $\int_1^3 R(t)\,dt$.

c) If Q is the function defined by $Q(x) = \int_1^x R(t)\,dt$, what is $Q'(3)$?

A **A**

Math 135 Final Exam

Wednesday, May 5, 1999, from 4 PM to 7 PM

NAME (*please print*): ______________________________

SIGNATURE: ______________________________

SECTION #: __________ LECTURER: ____________________

Do all problems, in any order.

Show all your work. Full credit may not be given for an answer alone.

You may use one sheet of notes and any standard calculator without a QWERTY keypad on this exam. You may use no other materials.

Problem Number	Possible Points	Points Earned:
1	10	
2	12	
3	18	
4	14	
5	16	
6	18	
7	12	
8	16	
9	18	
10	8	
11	8	
12	12	
13	12	
14	16	
15	10	
Total Points Earned:		

A **A**