Spring, 1996

Math 152 & 154 final exam review problems

The final exam will be cumulative, and all material on the syllabus which was covered by all lecturers should be considered as "eligible" for testing. The following material was not covered uniformly, and therefore will *not* be tested on the final: arc length (of either "standard" or parametric curves), surface area, moments, and centers of mass. Please be sure to include in your review for the final exam any previously suggested review problems and, especially, any previous exams. If we thought it was important before, we'll probably still believe it is important now! Note also that the problems below are in *no* particular order, and are intended *only* to be representative of the general type of problem which may be asked on the final.

1. Compute some integrals:

a) $\displaystyle\int_0^1 \arctan x \, dx$ b) $\displaystyle\int_0^{\sqrt{3}/2} x\sqrt{1-x^2}\, dx$ c) $\displaystyle\int_0^{\sqrt{3}/2} x^2\sqrt{1-x^2}\, dx$

2. a) Sketch the region in the first quadrant which is bounded by the curves $y = x^4$ and $y = x^2(1-x^2)$.

b) Find the area of the region sketched in a).

c) Find the volume of the solid which results when the region sketched in a) is revolved around the $x-$axis.

d) Find the volume of the solid which results when the region sketched in a) is revolved around the $y-$axis.

3. A cable that weighs 2 lbs/ft is used to lift 800 lb of coal up a mineshaft 500 ft deep. Find the work done.

4. Consider the definite integral $\displaystyle\int_0^1 (1+x)^{1/4}\, dx$.

a) Compute the definite integral using the Fundamental Theorem of Calculus.

b) Write out and compute the Trapezoid Rule approximation to this definite integral with $n = 4$ subdivisions.

c) Find n (the number of subdivisions) so that the Trapezoidal Rule estimate must be within 10^{-10} of the true value of the definite integral. (You may use the error bound $\dfrac{K(b-a)^3}{12n^2}$ where K is an overestimate of the magnitude of the second derivative.)

d) Write out and compute the Simpson's Rule approximation to this definite integral with $n = 4$ subdivisions.

e) Find n (the number of subdivisions) so that the Simpson's Rule estimate must be within 10^{-10} of the true value of the definite integral. (You may use the error bound $\dfrac{K(b-a)^5}{180n^4}$ where K is an overestimate of the magnitude of the fourth derivative.)

5. a) Write a power series centered at 0 which is equal to $\arctan x$ for x near 0.

b) Use your answer to a) to write a power series centered at 0 which is equal to $\arctan\left(x^3\right)$.

c) Use your answer to c) to write $\displaystyle\int_0^{1/2} \arctan\left(x^3\right) dx$ as a series.

d) Find a specific partial sum of this series which is equal to the definite integral with an error of at most 10^{-8}.

6. Compute some more integrals:

a) $\displaystyle\int_0^{\pi/4} \sec x\,(\tan x)^2\, dx$ b) $\displaystyle\int_0^{\pi/4} (\sec x)^2 \tan x\, dx$ c) $\displaystyle\int_0^1 \sqrt{x}e^{\sqrt{x}}\, dx$

7. Do these series converge? If yes, do they converge absolutely?

a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^2+1}$ b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \ln(n)}{\ln(n^2+1)}$ c) $\displaystyle\sum_{n=1}^{\infty} \frac{1-2^n}{3^n}$

8. a) What is $\displaystyle\lim_{n\to\infty} ne^{-n}$? b) What is $\displaystyle\lim_{n\to\infty} \left(\frac{n}{e}\right)^n \sqrt{2\pi n}$? c) What is $\displaystyle\lim_{n\to\infty} \frac{(n!)^2}{(2n)!}$?

9. a) Why does $\displaystyle\sum_{n=1}^{\infty} \frac{3}{\sqrt{n^3+7}}$ converge?

b) Find N so that $\displaystyle\sum_{n=1}^{N} \frac{3}{\sqrt{n^3+7}}$ is within .001 of the sum of the whole infinite series in a). Be sure to explain how you know that your answer is correct. Hint: compare what's *not* in the partial sum to an integral.

10. Sketch the graph of the following parametric curve, and eliminate the parameter to find the Cartesian equation of the curve:

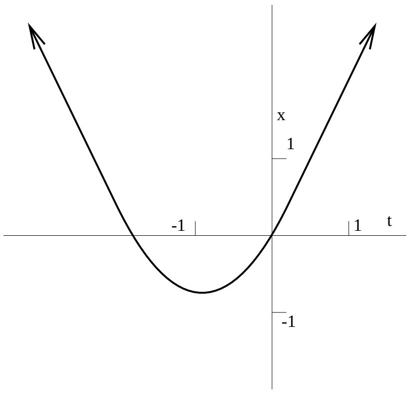
$$\begin{cases} x(t) = \dfrac{t^2}{1+t^2} \\ y(t) = \dfrac{t}{1+t^2} \end{cases}$$

11. a) Sketch the polar curves $r = 1+\theta$ and $r = 3\sin(2\theta)$ for $0 \le \theta \le \frac{\pi}{2}$.

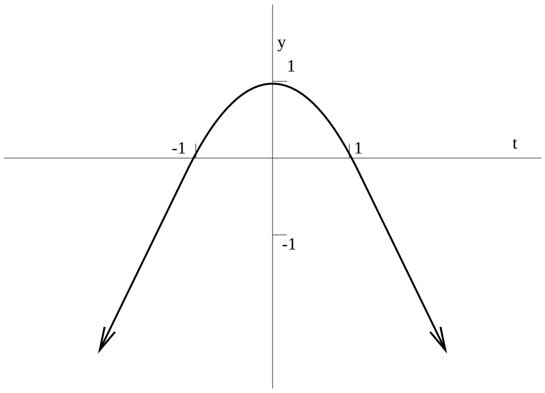
b) The curves intersect twice. Find the approximate values of θ at which they intersect.

c) What is the area of the region touching the origin and bounded by the two curves?

12. A curve is described by parametric equations $\begin{cases} x = f(t) \\ y = g(t) \end{cases}$ where the graphs of x as a function of t and y as a function of t are given below.



x=f(t)



y=g(t)

a) What is the (approximate) position of the point on the curve when $t = \frac{1}{2}$?

b) What is the (approximate) slope of the line tangent to the curve when $t = \frac{1}{2}$?

c) What is an equation of the line tangent to this curve when $t = \frac{1}{2}$ using the information you have obtained in a) and b)?

d) Sketch the curve as well as you can.

13. Find a positive number A so that if x is in the interval $[0, A]$ then $\left|\ln(1+x) - \left(x - \frac{1}{2}x^2 + \frac{1}{3}x^3\right)\right| < \frac{1}{100}$. Be sure to explain how you know that your answer is correct.

14. Describe a strategy for computing $e^{1/7}$ to ten decimal place accuracy. Be sure; explain why you're sure.

15. Solve:
a) $xy' = -y$, $x > 0$, $y(4) = 2$.
b) $y' = 1 + y^2$, $y\left(\frac{\pi}{3}\right) = 1$.

16. Just one more (and I'll even *give* you the answer):

$$\int_1^2 \frac{1}{x(x+1)(x+2)}\, dx = \ln\left(\sqrt{\frac{32}{27}}\right)$$

17. a) Compute $K_n = \int_0^1 x e^{-nx}\, dx$.

b) What is $\lim_{n\to\infty} K_n$?

18. Use Taylor's Theorem to find a polynomial $p(x)$ so that $|p(x) - \cos x| < .01$ for all x in the interval $[-1.5, 1.5]$. Be sure to explain how you know that your answer is correct.

in the longstanding tradition of this course
it
should be pointed out the the review roblems for the ginal soemtimes have ewwrs.