Math 151 Sample exam #2

1. Ant **A** is crawling up a vertical pole at .3 meters/minute. At the same time ant **B** is crawling away from the base of the pole on the horizontal ground at .4 meters/minute.

a) If θ is the angle that ant **B** sees between the base of the pole and ant **A**, if a is the distance from ant **A** to the base of the pole, and if b is the distance from ant **B** to the base of the pole, express θ as a function of a and b.

b) Use the information provided to compute θ and $\frac{d\theta}{dt}$ at the instant that ant **A** is 10 meters up the pole and ant **B** is 5 meters from the base of the pole. Be careful to include units with your answers!

2. Suppose that $C(8) = 4$ and $C'(x) = \frac{x^2 + 5x}{x + 1}$. If $B(x) = x^3$, and if $A(x) = C(B(x))$, what is $A(2)$? Write a formula for $A'(x)$ only in terms of x, and use this formula to find $A'(2)$. Use this to write an equation for the line tangent to the graph of $y = A(x)$ when $x = 2$.

3. Consider the function $T(x) = \frac{x}{e^{(x^2)}}$.

a) Sketch a graph of $y = T(x)$ in the window $-3 \le x \le 3$ and $-.5 \le y \le .5$.

b) Verify the geometric properties observed in the graph using the analytic tools of calculus:

> What are $\lim_{x\to\infty} T(x)$ and $\lim_{x\to-\infty} T(x)$ and why? Explain how your results apply to the graph sketched.
>
> Compute $T'(x)$. Find the roots of $T'(x) = 0$ algebraically. Where is $T'(x)$ positive and where is it negative? Explain how your results apply to the graph sketched.
>
> Compute $T''(x)$. Find the roots of $T''(x) = 0$ algebraically. Where is $T''(x)$ positive and where is it negative? Explain how your results apply to the graph sketched.

4. a) Sketch the graph of the function $f(x) = 1 - x - \sin(3x)$ on the interval $[0, 1]$ to verify that f has a root in this interval.

b) If Newton's method is applied to find a root of f, we obtain a sequence of numbers defined by a formula of the form $x_{n+1} = g(x_n)$. What is the function $g(x)$ when $f(x)$ is the function in part a)?

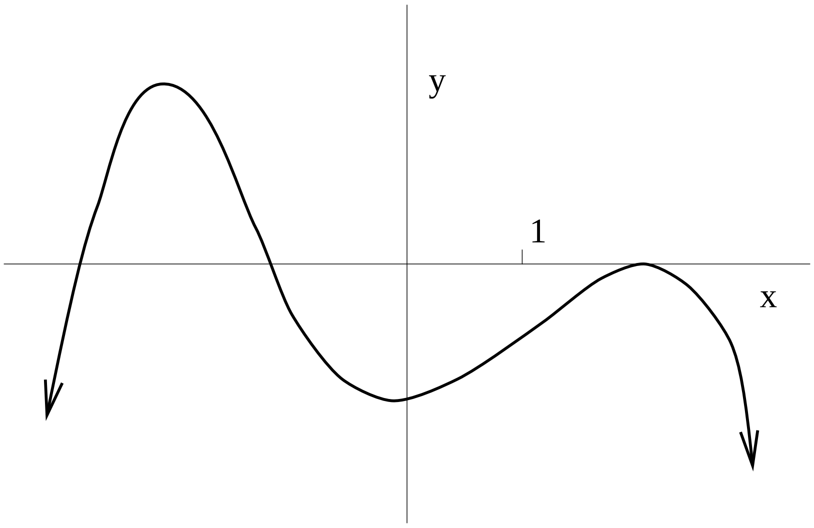
c) If the starting guess for Newton's method is taken to be 0.5, find the next two approximations x_1 and x_2.

d) Display graphically how Newton's method obtains the values x_1 and x_2. That is, on a single graph, sketch $f(x)$ and the two curves whose intersections with the x axis are the numbers x_1 and x_2.

5. a) Find the linear approximation of the function $f(x) = \sqrt{1-x}$ at $a = 0$.

b) Use the approximation determined in part (a) to approximate the number $\sqrt{.99}$.

6. Below is a graph of the **derivative** of F. You may wish to label significant points on this graph to help you answer what is asked below.



Graph of $y = F'(x)$

a) Sketch a graph of $y = F''(x)$, the **second** derivative of F.

b) Suppose you also know that $F(1) = 1$. Sketch a graph of $y = F(x)$ as well as you can. Label each of the following: any local extrema (being careful to tell which are local maxima and which are local minima) and any points of inflection.

c) In which intervals is F increasing? In which intervals is F decreasing? In which intervals is F concave up? In which intervals is F concave down?

7. Suppose Q is a function satisfying the following differential equation:

$$Q'(x) = 3Q(x)^2 + 2Q(x) + 1$$

and we know that $Q(4) = 5$. What is $Q'(4)$?

a) If R is a function *inverse* to Q, find an equation for $R'(w)$ in terms of w only. (Hint: what is $Q(R(w))$? Differentiate the resulting equation.)

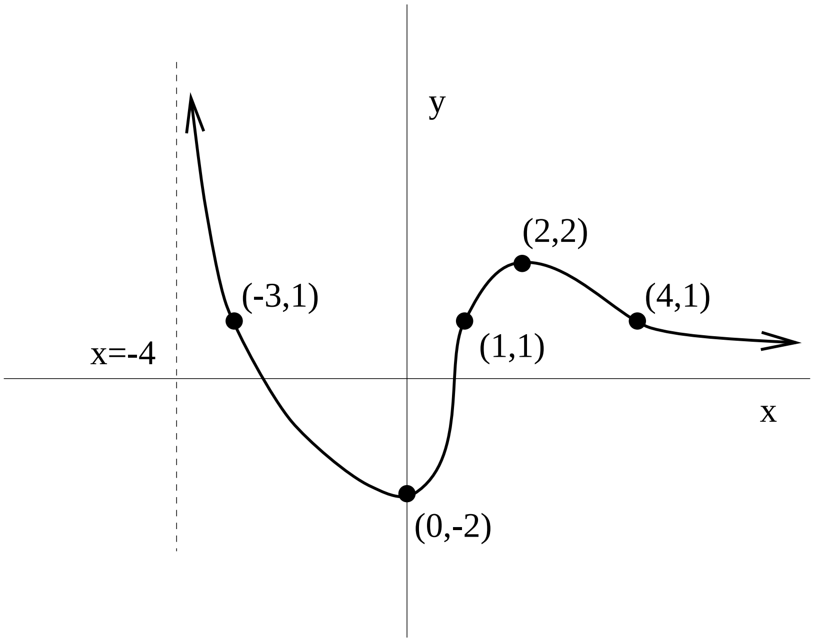
b) What are $R(5)$ and $R'(5)$?

8. Find the line tangent to the curve defined by the equation

$$\ln(xy) + 2x - y + 1 = 0$$

at the point $\left(\frac{1}{2}, 2\right)$.

9. Below is a graph of $y = H(x)$.



Graph of $y = H(x)$

Here is a table of some values of H and of its derivative, H':

x	$H(x)$	$H'(x)$
-3	1	-4
0	-2	0
1	1	2
2	2	0
4	1	-2

a) What is the largest interval containing $x = 1$ in which H is one-one? Suppose that G is the function inverse to H on this interval. Sketch a graph of G as well as you can. What is $G(1)$ and what is $G'(1)$?

b) What is the largest interval containing $x = 4$ in which H is one-one? Suppose that K is the function inverse to H on this interval. Sketch a graph of K as well as you can. What is $K(1)$ and what is $K'(1)$?

10. Suppose $p(x) = x - \sqrt{x^2 - 3x}$

a) Give a table of values of $p(x)$ for $x = 500,\ 1,000,\ 1,500$, and $2,000$.

b) Find $\lim_{x \to +\infty} p(x)$.

11. Let $q(x) = (1 + 2x)^{1/x}$.

a) Give a table of values of $q(x)$ for $x = -.002,\ -.001,\ .001$ and $.002$.

b) Find $\lim_{x \to 0} q(x)$.

12. Suppose f is a function defined for all x. Assume also that f' and f'' exist for all x. (This means, in particular, that the graph of f has no corners.) Assume that:

$$f(0) = -2, f(2) = 2, f(4) = 1$$

$$f'(0) = f'(2) = f'(4) = 0$$

$$f'(x) < 0 \text{ for } x < 0 \text{ and } 2 < x < 4$$

$$f'(x) > 0 \text{ for } 0 < x < 2$$

$$f''(x) < 0 \text{ for } x < -1 \text{ and } 1 < x < 3$$

$$f''(x) > 0 \text{ for } -1 < x < 1 \text{ and } 3 < x$$

a) Find all local minima and maxima inside the interval $(-\infty, 3)$.

b) Find all inflection points.

c) Sketch the part of the graph that lies over the interval $(-\infty, 4]$.

d) Is it possible that $f'(x) < 0$ for all $x > 4$? Explain your answers (in complete sentences.) Recall that $f'(4) = 0$ and that the graph of f cannot have corners.

13. If $S(1) = 10$ and $S'(x) \geq 2$ for $1 \leq x \leq 4$, how small can $S(4)$ possibly be?

14. Suppose $F(x) = x \arctan\left(\frac{1}{x}\right)$ if $x \neq 0$ and $F(0) = 0$.

a) Sketch the graph of F.

b) Is F continuous at 0? Justify your answer using the definition of continuity.

c) Is F differentiable at 0? Justify your answer using the definition of differentiability.