Lecture 8.

Systems of differential equations

(Sect. 4.6)

$$\begin{cases} x' = x + y & x(0) = 1 \\ y' = x - y & y(0) = -1 \end{cases}$$

$x(t), y(t)$

$X = \mathcal{L}x,\ Y = \mathcal{L}y$ $\quad X(s), Y(s)$

$$\begin{cases} sX - x(0) = X + Y \\ sY - y(0) = X - Y \end{cases}$$

$$\begin{cases} (s-1)X - Y = 1 \\ -X + (s+1)Y = -1 \end{cases}$$

$$(s^2-1)X - X = s+1-1$$

$$(s^2-2)X = s$$

$$X = \frac{s}{s^2-2}, \quad x = \mathcal{L}^{-1}(X) = \boxed{\cosh(\sqrt{2}t)}$$

$$y = x' - x = \boxed{\sqrt{2}\sinh(\sqrt{2}t) - \cosh(\sqrt{2}t)}$$

$(\sinh)'(t) = \cosh t$

$(\cosh)'(t) = \sinh t$

$x(0) = 1, y(0) = -1$

Another solution.

$$X = \frac{s}{s^2-2} = \frac{a}{s-\sqrt{2}} + \frac{B}{s+\sqrt{2}} = \frac{1}{2}\left(\frac{1}{s-\sqrt{2}} + \frac{1}{s+\sqrt{2}}\right)$$

$$a+B=1$$
$$\sqrt{2}(a-B)=0 \qquad a=B=\frac{1}{2}$$

$$x(t) = \frac{1}{2}\left(e^{\sqrt{2}t} + e^{-\sqrt{2}t}\right)$$

$$y(t) = x'(t) - x(t) =$$
$$= \frac{1}{2}\left((\sqrt{2}-1)\,e^{\sqrt{2}t} - (\sqrt{2}+1)\,e^{-\sqrt{2}t}\right).$$