MATH 421.04 ADVANCED CALCULUS FOR ENGINEERING. FALL 2014. QUIZ 1

1. (75 points)

a) Find given Laplace transforms

$\mathcal{L}\{(1-e^t)^2\}$;

$\mathcal{L}\{e^{-2t}t^3\}$;

$\mathcal{L}\{\mathcal{U}(t-2)\sin 2t\}$.

b)Give the formula representing $\mathcal{L}f'$ through $\mathcal{L}f$ and prove it.

a) 1. $\mathcal{L}\{(1-e^t)^2\} = \mathcal{L}(1-2e^t+e^{2t}) =$

$= \frac{1}{s} - \frac{2}{s-1} + \frac{1}{s-2}.$

2. $\mathcal{L}(e^{-2t}t^3) = \frac{6}{(s+2)^4}.$

3. $\mathcal{L}(\mathcal{U}(t-2)\sin 2t) = e^{-2s}\left(\frac{2\cos 4}{s^2+4} + \frac{\sin(4)\cdot s}{s^2+4}\right).$

$a=2,\ g(t)=\sin 2t$

$g(t+2) = \sin(2t+4) = \sin 2t\cos 4 + \cos 2t \sin 4$

b) $\mathcal{L}f' = s\mathcal{L}f - f(0)$

$\mathcal{L}f' = \int_0^\infty e^{-st} f'(t)\,dt = e^{-st}f(t)\Big|_0^\infty + s\int_0^\infty e^{-st}f(t)\,dt$

$= -f(0) + s\mathcal{L}f.$

2. (75 points) Find given inverse transforms

$$\mathcal{L}^{-1}\left(\frac{s}{(s-1)(s+2)(s-3)}\right) = -\frac{e^t}{6} - \frac{2e^{-2t}}{15} + \frac{3}{10}e^{3t};$$

$$\mathcal{L}^{-1}\left(\frac{1}{(s-4)^2}\right) = e^{4t}t;$$

$$\mathcal{L}^{-1}\left(\frac{1}{s^2-s+5}\right) = \mathcal{L}^{-1}\left(\frac{1}{(s-\frac{1}{2})^2+\frac{19}{4}}\right) = \frac{2}{\sqrt{19}}e^{t/2}\sin\left(\frac{\sqrt{19}}{2}t\right);$$

$$\mathcal{L}^{-1}\left(\frac{e^{-2s}}{s^2+4}\right) = \frac{1}{2}u(t-2)\sin(2(t-2)).$$

$$\frac{s}{(s-1)(s+2)(s-3)} = \frac{a}{s-1} + \frac{B}{s+2} + \frac{C}{s-3}$$

$$a(s+2)(s-3) + B(s-1)(s-3) + C(s-1)(s+2) = s$$

$$\begin{array}{l|ll} s=1 & -6a = 1 & a = -\frac{1}{6} \\ s=-2 & 15B = -2 & B = -\frac{2}{15} \\ s=3 & 10C = 3 & C = \frac{3}{10}. \end{array}$$

3. (50 points) Solve given initial-value problem using the Laplace transform
$y' - 2y = e^{3t}, y(0) = 2;$

$Y = \mathcal{L}y$

$(s-2)Y = \frac{1}{s-3} + y(0)$

$(s-2)Y = \frac{1}{s-3} + 2 = \frac{2s-5}{(s-2)(s-3)}$

$\frac{2s-5}{(s-2)(s-3)} = \frac{a}{s-2} + \frac{B}{s-3}$

$a(s-3) + B(s-2) = 2s-5$

$s=2 \quad -a=-1, \quad \boxed{a=1}$

$s=3 \quad \boxed{B=1}$

$y = ae^{2t} + Be^{3t} = e^{2t} + e^{3t}, \quad y(0) = 2,$

e^{-3s} 7.1

3. $\mathcal{L}^{-1}\left(e^{-3s}\left(\frac{s^2+1}{4s^2-1}\right)\right)$ $\frac{1}{4}\mathcal{L}^{-1}\left(e^{-3s}+\frac{5/4}{s^2-\frac{1}{4}}\right)$

$$\mathcal{L}^{-1}\left(\frac{s^2+1}{4s^2-1}\right)=\mathcal{L}^{-1}\left(\frac{1}{4}\left(1+\frac{5/4}{s^2-\frac{1}{4}}\right)\right)$$ $k=\frac{1}{2}$

$$\boxed{=\frac{1}{4}\left(\delta(t-3)+\frac{5}{2}\mathcal{U}(t-3)\sin\left(\frac{t-3}{2}\right)\right)}$$

we dont need $\mathcal{U}(t-3)$

4. $y''+y=\delta(t-\pi),\ y(0)=1,\ y'(0)=0$

$$(s^2+1)Y=e^{-s\pi}+sy(0)+y'(0)$$

$$(s^2+1)Y=e^{-s\pi}+s$$

$$Y=\frac{e^{-s\pi}+s}{s^2+1}$$

$$y=\cos t+\mathcal{U}(t-\pi)\sin(t-\pi)=\cos t-\mathcal{U}(t-\pi)\sin t.$$