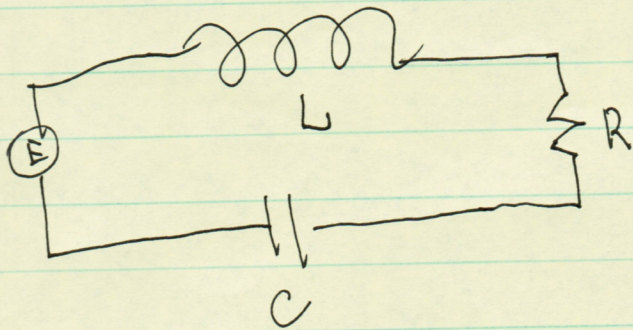


Kirchhoff's 2nd law



Impressed voltage $E(t)$
current $i(t)$

Voltage drop across

- inductor

$$L \frac{di}{dt}$$

- resistor

$$R i(t)$$

- capacitor

$$\frac{1}{C} \int_0^t i(\tau) d\tau$$

(L, R, C) are constant

$$L \frac{di}{dt} + R i(t) + \frac{1}{C} \int_0^t i(\tau) d\tau = E(t).$$

Lecture 6.

(Section 4.5) 6.1

 δ -function. (The Dirac Delta function)

$$\delta(t) = \begin{cases} \infty & t=0 \\ 0 & t \neq 0 \end{cases} \quad \int_{-\infty}^{\infty} \delta(t) dt = 1$$



It's a "generalized function"

(corresponds to the unit impulse of the duration 0)

Properties:

$$1) \int_a^b \delta(t) dt = \begin{cases} 1 & a < 0 < b \\ 0 & \text{otherwise} \end{cases}$$

$$2) \int_{-\infty}^{\infty} \delta(t) f(t) dt = f(0)$$

$$3) \int_{-\infty}^{\infty} \delta(t-a) f(t) dt = f(a)$$

$$4) a > 0 \quad \mathcal{L}(\delta(t-a)) = \int_0^{\infty} e^{-ts} \delta(t-a) dt = e^{-as}$$

$$\boxed{\mathcal{L}(\delta(t-a)) = e^{-as}, \quad a \geq 0}$$

$$\mathcal{L}(\delta(t)) = 1$$

$$\Rightarrow \text{Table 1,} \quad \mathcal{L}^{-1}(e^{-as}) = \delta(t-a)$$

$$\mathcal{L}^{-1}(1) = \delta(t), \quad a \geq 0$$

Now we can compute $\mathcal{L}^{-1}\left(\frac{P(s)}{Q(s)}\right)$ if
 $\deg P = \deg Q$.

Examples.

$$1. \mathcal{L}^{-1}\left(\frac{2s^2 - 4s + 9}{s^2 - 2s + 5}\right) =$$

$$= \mathcal{L}^{-1}\left(2 - \frac{1}{s^2 - 2s + 5}\right)$$

$$= 2\delta(t) - \mathcal{L}^{-1}\left(\frac{1}{(s-1)^2 + 4}\right)$$

$$= 2\delta(t) - \frac{e^t}{2} \sin 2t.$$

Separate the integer.
 ii

the fraction
 of highest coef.

$$2. \mathcal{L}^{-1}\left(e^{-2s} \frac{s^2 - 1}{s^2 + 4}\right) = \delta(t-2) - \frac{5}{2} u(t-2) \sin(2(t-2))$$

$$\mathcal{L}^{-1}\left(\frac{s^2 - 1}{s^2 + 4}\right) = \mathcal{L}^{-1}\left(1 - \frac{5}{s^2 + 4}\right) = \delta(t) - \frac{5}{2} \sin 2t$$