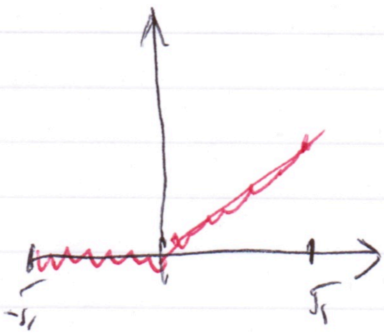
Lecture 13.

Ex. $f(x) = \cancel{x} \frac{1}{2}(x+|x|) = \begin{cases} x, & 0 < x < \pi \\ 0, & -\pi < x < 0 \end{cases}$



Take the sum of Fourier series:

Each f on $[-p,p]$ can be present as

$$f(x) = f_{even}(x) + f_{odd}(x)$$

$$f_{even}(x) = \frac{f(x)+f(-x)}{2}$$

$$f_{odd}(x) = \frac{f(x)-f(-x)}{2}$$

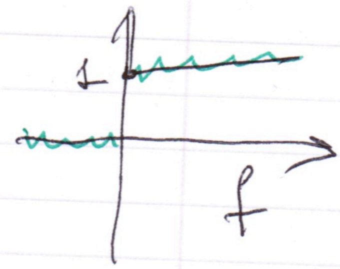
Verify for our example

$$f(x) = \frac{\pi}{2} - \frac{4}{\pi}\sum_{k=0}^{\infty} \frac{\cos((2k+1)x)}{(2k+1)^2} = \frac{|x|}{2}$$

$$+ 2\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\sin nx}{n}, = \frac{x}{2}$$

Around HA on 12.2.

Pr. 1. $f(x) = \begin{cases} 0, & -\pi < x < 0 \\ 1 & 0 < x < \pi \end{cases}$



$$f(x) = \frac{1}{2} \left(\operatorname{sgn} x + 1 \right)$$

$$f(x) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \cancel{\text{sgn}} \sin nx}{\cancel{4} n}$$

Pr. 9.

13.6.

$$f_1(x) = \cancel{f(x)}\, x^2, \quad -\pi < x < \pi \qquad p = \pi$$

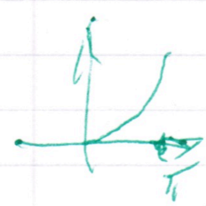
$$\underset{n\neq 0}{a_n} = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2 \cos nx \, dx = \underbrace{\frac{1}{\pi}\,\frac{x^2 \sin nx}{n}\Big|_{-\pi}^{\pi}}_{=0}$$

$$- \frac{2}{\pi n} \int_{-\pi}^{\pi} x \sin nx = \frac{2(-1)^{n+1}}{n}\pi \quad \text{(old computation)}$$

$$a_n = \frac{4(-1)^n}{n^2}, \quad n \neq 0$$

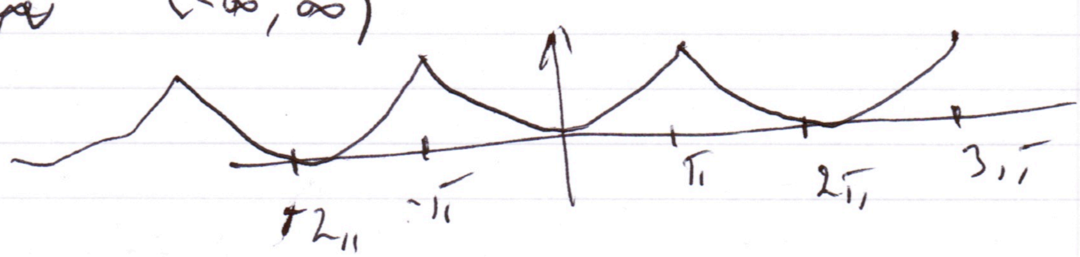
$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2 \, dx = \frac{2\pi^2}{3}$$

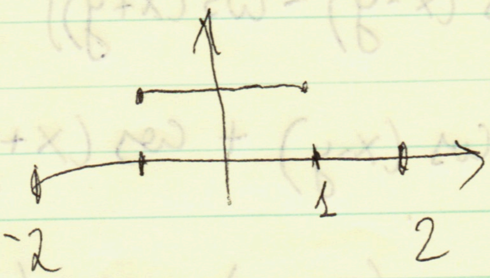
$$\boxed{x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos nx. \quad -\pi < x < \pi}$$



Corollary. It's the Cosine Fourier series on $[0, \pi]$

On $(-\infty, \infty)$





$[-2, 2]$

$p = ?$

$x = 0$

$$0 = \frac{\pi^2}{3} + 4 \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$$

$$\frac{\pi^2}{12} = \sum \frac{(-1)^{n+1}}{n^2} = 1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + -.$$

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} \cdots$$

$$\frac{\pi^2}{8} = 1 + \frac{1}{9} + \frac{1}{25} + \frac{1}{36} + \cdots$$

$$\frac{\pi^2}{12} = 1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \cdots$$

Cosine and Sine Fourier Series.

Sect. 12.3

- $\left\{ \cos \frac{n\pi x}{p}, \sin \cancel{mx} \frac{m\pi x}{p} \right\}$ is complet on $[-p, p]$ 5

Each of

$$\left\{ \cos \frac{n\pi x}{p} \right\} \text{ and } \left\{ \sin \frac{m\pi x}{p} \right\}$$

are complet on $[0, p]$.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{p} \quad \text{[cosine series]}$$

$$= \sum_{m=1}^{\infty} b_m \sin \frac{m\pi x}{2} \quad \text{[sine series]}$$

$$a_n = \frac{2}{p} \int_0^p f(x) \cos \frac{n\pi x}{p} \, dx$$

$$b_m = \frac{2}{p} \int_0^p f(x) \sin \frac{m\pi x}{p} \, dx$$

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Cosine and Sine Fourier Series

Sect.

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Cosine and Sine Fourier Series

Sect. 12.3

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$$a_n = \frac{2}{p}\int_0^p f(x)\cos\frac{n\pi x}{p}\,dx$$

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13.8

6

Fourier S. on $[-p,p]$ for even functions $\Longleftrightarrow$ Cosine F. Series on $[0,p]$.

-11- for odd functions $\Longleftrightarrow$ Sine F series on $[0,p]$

The same function $f(x)$ on $[0,p]$ can be expand at both Cosine and Sine Fourier series.

Examples.

$p=\pi$ $\quad f(x)=x$

Even extens $|x|$

Odd one x

Cosine F. series

$$x=\frac{\pi}{2}-\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n-1}{n^2}\cos nx$$

$0<x<\pi$

Sine F. series

$$x=2\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\sin nx.$$

~~13.5.~~

~~13.5~~

Extensions of series on $(-\infty, \infty)$ are different: