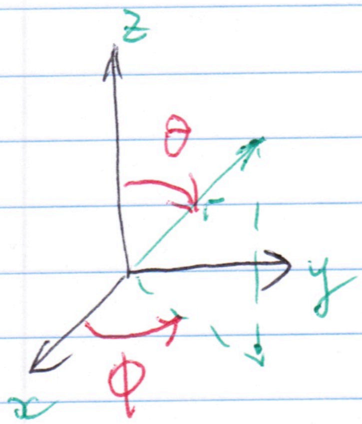
Spherical Coordinates.



$$x = r \sin\theta \cos\phi$$
$$y = r \sin\theta \sin\phi$$
$$z = r \cos\theta.$$

$$u(r,\theta,\phi)$$

$$\Delta u = u_{rr} + \frac{2}{r} u_r + \frac{1}{r^2 \sin^2\theta} u_{\phi\phi} + \frac{1}{r^2} u_{\theta\theta} + \frac{\cot\theta}{r^2} u_\theta .$$

Simplifications.

u is independent of ϕ.

$u(r,\theta)$ at the ball $0<r<c$.

$$u_{rr} + \frac{2}{r} u_r + \frac{1}{r^2} u_{\theta\theta} + \frac{\cot\theta}{r^2} u_\theta = 0, \quad 0<r<c, \quad 0<r<\pi$$

$u = R(r)\,\Theta(\theta)$

$$\frac{r^2 R'' + 2r R'}{R} = \frac{\Theta'' + \cot \Theta'}{\Theta} = \lambda$$

(1) $r^2 R'' + 2r R' - \lambda R = 0$

(2) $\boxed{\sin\theta\, \Theta'' + \cos\theta\, \Theta' + \lambda \sin\theta\, \Theta = 0.}$

at (2) the substitution $x = \cos\theta$:

$$(1-x^2)\frac{d^2\Theta}{dx^2} - 2x\frac{d\Theta}{dx} + \lambda\Theta = 0, \quad -1 < x < 1$$

Legendre's equation!

$$\Theta_n(\theta) = P_n(\cos\theta), \quad \lambda_n = n(n+1), \quad n = 0, 1, 2, \ldots$$

$$R_n(r) = a_n r^n + b_n r^{-(n+1)}$$

(a version of Cauchy-Euler eq.

$$u(r,\theta) = \sum_{n=0}^{\infty} \left(a_n r^n + b_n r^{-(n+1)}\right) P_n(\cos\theta).$$

Regularity at the origin $\Rightarrow b_n \equiv 0$.