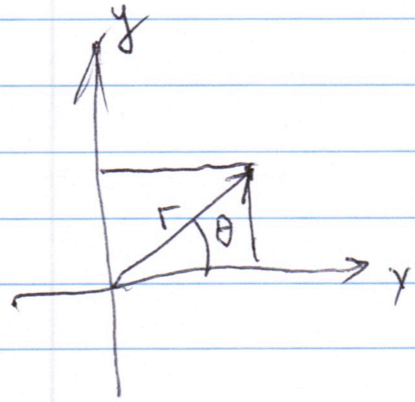
Polar coordinates. (Sect. 14.1).

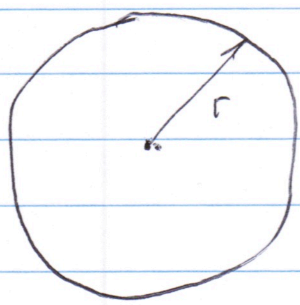


$$x = r\cos\theta \qquad r = \sqrt{x^2+y^2}$$
$$y = r\sin\theta \qquad \tan\theta = \frac{y}{x}.$$

Laplacian in Polar coordinates.

(1) $$\Delta u = \frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}.$$

Aim: Problems in curved domain.



$\Delta u = 0.$

Separation of variables in polar coordinates.

$$u(r,\theta) = R(r)\Theta(\theta)$$

(2) $$R''\Theta + \frac{1}{r}R'\Theta + \frac{1}{r^2}R\Theta'' = 0$$

$$\frac{r^2R'' + rR'}{R} = -\frac{\Theta''}{\Theta} = \lambda$$

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(3) $\Theta'' + \lambda \Theta = 0, \quad 0 < \theta < 2\pi$

$\Theta(\theta) = \Theta(\theta + 2\pi), \; \Theta'(\theta) = \Theta'(\theta + 2\pi).$

Periodic ~~co~~ condition – Singular Sturm-Liouville Problem.

$\lambda \geq 0 \quad \Theta_0 = c$

$$\Theta_n(\theta) = c_1 \cos(n\theta) + c_2 \sin(n\theta), \; \lambda = n^2$$
$n = 0, 1, 2,$

$R(r)$

(4) $$r^2 R'' + r R' - \lambda R = 0.$$

Cauchy-Euler equation

$\lambda = \alpha^2 \qquad R_\alpha(r) = c_1 r^\alpha + c_2 r^{-\alpha}, \; \alpha > 0,$

$R_0(r) = c_1 + c_2 \ln r, \quad \alpha = 0$

λ is joint at (2): $\lambda = n^2, \; n = 0, 1, 2, \dots$

$$u_n(r,\theta) = R_n(r)\,\Theta_n(\theta)$$
$$= (c_n r^n + d_n r^{-n})(a_n \cos(n\theta) + b_n \sin(n\theta))$$
$n = 1, 2$

$$u_0(r,\theta) = c_0 + d_0 \ln r.$$