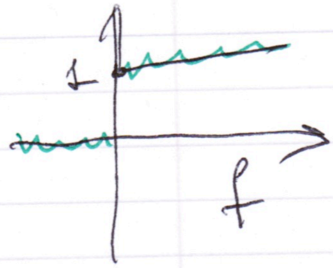
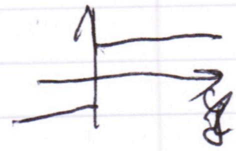
Lecture 14.

Around HA on 12.2.

Pr. 1. $f(x) = \begin{cases} 0, & -\pi < x < 0 \\ 1 & 0 < x < \pi \end{cases}$



$$f(x) = \frac{1}{2} \left(\text{sgn } x + 1 \right)$$

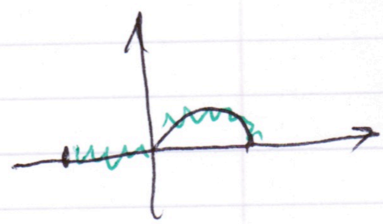


sgn x

$$f(x) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \sin nx}{n}$$

Pr. 9.

$$f(x) = \begin{cases} 0, & \cancel{0 < x < \pi} \quad -\pi < x < 0 \\ \sin x, & 0 < x < \pi \end{cases}$$

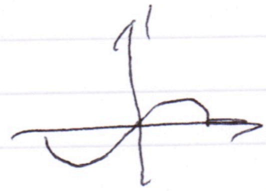


$$f(x) = \frac{1}{2}\left(\sin x + |\sin x| \right).$$

$$= \frac{\sin x}{2} + \frac{1}{\pi}\left(1 - \sum_{\text{even } n} \frac{\cos nx}{n^2 - 1} \right)$$

$\frac{1}{2}|\sin x|$

$\frac{1}{2} \sin x$



Examples of Cosine and Sine Series:

$[0,\ell]$

$$x = \frac{2\ell}{\pi}\left(\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\sin nx}{n}\right.$$

[corresponds $f(x)=x$ on $[-\ell,\ell]$)

$$x = \frac{\ell}{2} - \frac{4\ell}{\pi^2}\sum_{k=0}^{\infty} \frac{\cos\left((2k+1)\frac{\pi x}{\ell}\right)}{(2k+1)^2}$$

(corresponds $|x|$ on $[-\ell,\ell]$

$$= \frac{\ell}{2} - \frac{2\ell}{\pi^2}\sum_{n=1}^{\infty} \frac{(-1)^n - 1}{n^2}\cos\left(\frac{n\pi x}{\ell}\right)$$

$$1 = \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{1-(-1)^n}{n}\sin\left(\frac{n\pi x}{\ell}\right)$$

$$= \frac{4}{\pi}\sum_{k=0}^{\infty} \frac{1}{2k+1}\sin\left(\frac{(2k+1)\pi x}{\ell}\right).$$

corresponds $\operatorname{sgn} x$ on $[-\ell,\ell]$

$1=1$

$[0, \pi]$

$$\cos x = \frac{4}{\pi} \sum_{n \text{ even}} \frac{n}{n^2 - 1} \sin nx,$$

$$\sin x = \frac{2}{\pi} \left(1 - 2 \sum_{n \text{ even}} \frac{\cos nx}{n^2 - 1} \right),$$

Substitution $x=0$ at the ~~Co~~sine series for $\sin x$.

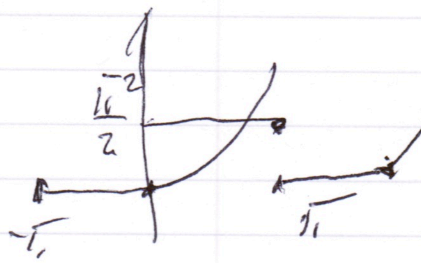
$$1 = \sum_{\text{even } n} \frac{1}{n^2-1} = \frac{1}{3} + \frac{1}{8} + \frac{1}{15} + \cdots$$

Equivalent

$$1\,\cancel{\tfrac{1}{2}} = \sum_{\text{even}} \left(\frac{1}{n-1} - \frac{1}{n+1} \right) = \left(1 - \cancel{\frac{1}{3}}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{5}}\right) + \cdots$$

Problem 5.

$$f(x) = \begin{cases} x^2, & 0 < x < \pi \\ 0, & -\pi < x < 0 \end{cases}$$



$$f(x) = \frac{\pi^2}{6} + 2\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos nx$$
$$+ \sum_{m=1}^{\infty} \left(\frac{\pi(-1)^{m+1}}{m} + \frac{2\left((-1)^m - 1\right)}{\pi m^3} \right) \sin mx$$

#17.

$x=0$ $$\frac{\pi^2}{12} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} = 1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2}$$

#17

$x = \pi$, $\quad \dfrac{\pi^2}{2} = \dfrac{\pi^2}{6} + 2\sum \dfrac{1}{n^2}$

$\cos n\pi = (-1)^n$

$$\sum \frac{1}{n^2} = \pi^2\left(\frac{1}{4} - \frac{1}{12}\right) = \frac{\pi^2}{6}$$

$$\frac{\pi^2}{6} = \sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \cdots$$

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} \ldots$$

X

$$\frac{\pi^2}{8} = 1 + \frac{1}{9} + \frac{1}{25} + \frac{1}{49} + \ldots$$

1x1

$$\frac{\pi^2}{12} = 1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2}$$

$$\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \ldots$$