MATH 300. INTRODUCTION TO
MATHEMATICAL REASONING.
FALL 2015.
QUIZ 4

1. (50 points) Prove the following Tautologies

$$(P \Rightarrow Q) \Leftrightarrow (\sim Q \Rightarrow \sim P);$$

$$(P \Rightarrow Q) \Leftrightarrow ((P \wedge \sim Q) \Rightarrow \mathbf{F}),$$

where **F** is a Contradiction.

Explain how we apply them at Proofs by Contradiction.

Compare when 2 sides of the biconditional are False

a) Left: $P \Rightarrow Q$ is False iff P-T, Q-F

Right: $\sim Q$ is True, $\sim P$ is False $\Leftrightarrow$

Q is False, P is True

The same.

Reduction to Contradiction: If Q is False then P is False

b) Right is False iff $P \wedge \sim Q$ is True so iff

P is True and $\sim Q$ is True (s.t. Q is False).

In proof by contradiction we suppose tat P-True, Q-false and received any Contradiction

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Another solution:

a) $(P \Rightarrow Q) \Leftrightarrow (\sim P \vee Q)$

$(\sim Q \Rightarrow \sim P) \Leftrightarrow$ ~~$(Q \vee \sim$~~ $(Q \vee \sim P)$

The same

b) $((P \wedge \sim Q) \Rightarrow F) \Leftrightarrow \sim(P \wedge \sim Q) \vee F$

$\Leftrightarrow \sim P \vee Q.$

$R \vee F \Leftrightarrow R$

2. (50 points) Prove that

1) A natural number is odd iff it can be present as the difference of squares of consequent natural numbers.

2) Natural numbers a, b have in division on c the same remainder iff their difference is multiple to c.

1) $P \Leftrightarrow Q$ $P:$ $n = 2k+1$

$Q:$ $n = (m+1)^2 - m^2$

$Q \Leftrightarrow n = 2m+1.$

2) $P \Leftrightarrow Q$

$P:$ $a = cq_1 + r \wedge b = cq_2 + r$

$Q:$ $c \mid (a-b) \Leftrightarrow a - b = cq_3.$

$P \Rightarrow$ $a - b = c(q_1 - q_2) \Rightarrow a - b = c\, q_3 \wedge q_3 = q_1 - q_2 \Leftrightarrow Q$

$Q \Rightarrow a - b = cq_3 \Rightarrow (a = cq_1 + r \Rightarrow \cancel{b = a - c(q_1 - q_2) =}$

$\cancel{= cq_2 + r.}$ $\blacksquare$

$= b = a - (a-b) = cq_1 + r - cq_3 =$

$= c(q_1 - q_3) + r = cq_2 + r.$