

MATH 300:01. INTRODUCTION
TO MATHEMATICAL REASONING.

FALL 2015.

QUIZ 3

1. (75 points) Determine whether each of the following is a tautology, contradiction, or neither. Don't use True tables but transform formulas

$$P \vee (\sim P \wedge Q) \Leftrightarrow P \vee Q;$$

$$\sim (P \Rightarrow Q) \Leftrightarrow (P \wedge \sim Q);$$

a) $P \vee (\sim P \wedge Q) \stackrel{(P \wedge \sim P) \Rightarrow Q}{\Leftrightarrow}$

distribution

$$\Leftrightarrow \underbrace{(P \vee \sim P)}_{\textcircled{T}} \wedge (P \vee Q) \Leftrightarrow P \vee Q;$$

$$\textcircled{T} \wedge R \Leftrightarrow R;$$

b) $\sim (P \Rightarrow Q) \Leftrightarrow \sim (\sim P \vee Q) \Leftrightarrow P \wedge \sim Q$ de Morgan

c) $P \wedge \sim P$ - contradiction $\textcircled{F}$ (identical F)

$$\textcircled{F} \Rightarrow Q \Leftrightarrow \textcircled{T}$$

2. (25 points) Prove that $(x^2 - 1) < 0$ iff $-1 < x < 1$. (x are real numbers).

1st way:

$$(x^2 - 1) < 0 \Leftrightarrow (x-1)(x+1) < 0 \Leftrightarrow$$

$$((x-1) < 0 \wedge (x+1) > 0) \vee ((x-1) > 0 \wedge (x+1) < 0) \Leftrightarrow$$

$$(x < 1 \wedge x > -1) \vee (x > 1 \wedge x < -1) \Leftrightarrow$$

$$(-1 < x < 1) \vee (\text{F}) \Leftrightarrow (-1 < x < 1). \quad \square$$

2nd way:

Equiv. $x^2 < 1 \Leftrightarrow |x|^2 < 1 \Leftrightarrow \cancel{x > 0 \wedge x < 1}$

$$|x| < 1 \Leftrightarrow \cancel{0 \leq x < 1 \vee -1 < x \leq 0}$$

$$(x \geq 0 \wedge x < 1) \vee (x \leq 0 \wedge x > -1)$$