

MATH 300:01. INTRODUCTION  
TO MATHEMATICAL REASONING.  
FALL 2015.  
QUIZ 2

1. (50 points) Give convenient (useful) negation of the formula

$$\sim (P \wedge Q) \vee (\sim P \wedge \sim Q).$$

$$\sim (\sim (P \wedge Q) \vee (\sim P \wedge \sim Q)) \equiv$$

De Morgan +  
double negation

$$(P \wedge Q) \wedge (P \vee Q) \equiv$$

De Morgan

$$P \wedge Q \wedge (P \vee Q) \equiv$$

associativity

$$P \wedge Q.$$

$$P \wedge (P \vee Q) \equiv P.$$

2. (50 points) Let  $f(P_1, P_2, P_3)$  is a connective which is False iff an even number of the propositions  $P_i$  is False.

a) Construct True table and represent it by a formula through conjunctions, disjunctions and negations.

b) Find a similar formula with only conjunctions and negations.

Even number - F  $\Leftrightarrow$  Odd number is T

| $P_1$ | $P_2$ | $P_3$ | $f(P_1, P_2, P_3)$ |
|-------|-------|-------|--------------------|
| T     | T     | T     | F                  |
| T     | T     | F     | T                  |
| T     | F     | T     | T                  |
| T     | F     | F     | F                  |
| F     | T     | T     | T                  |
| F     | T     | F     | F                  |
| F     | F     | T     | F                  |
| F     | F     | F     | T                  |

$$A \vee B = \sim(\sim A \wedge \sim B)$$

$$f = (P_1 \wedge P_2 \wedge \sim P_3) \vee (P_1 \wedge \sim P_2 \wedge P_3) \vee (\sim P_1 \wedge P_2 \wedge P_3) \vee (\sim P_1 \wedge \sim P_2 \wedge \sim P_3)$$

$$= \sim[\sim(P_1 \wedge P_2 \wedge \sim P_3) \wedge \sim(P_1 \wedge \sim P_2 \wedge P_3) \wedge \sim(\sim P_1 \wedge P_2 \wedge P_3) \wedge \sim(\sim P_1 \wedge \sim P_2 \wedge \sim P_3)]$$

MATH 300:02. INTRODUCTION  
TO MATHEMATICAL REASONING.  
FALL 2015.  
QUIZ 2

1. (50 points) Give convenient (useful) negation of the formula

$$(\sim P \vee \sim Q) \wedge \sim (P \vee Q).$$

$$\sim((\sim P \vee \sim Q) \wedge \sim (P \vee Q))$$

De Morgan +  
double negation

$$\equiv \sim(\sim P \vee \sim Q) \vee (P \vee Q)$$

$$\equiv (P \wedge Q) \vee P \vee Q$$

De Morgan +  
associativity

$$\equiv P \vee Q$$

$$P \wedge (P \vee Q) \equiv P$$

2. (50 points) Let  $f(P_1, P_2, P_3)$  is a connective which is False iff an odd number of the propositions  $P_i$  is False.

a) Construct True table and represent it by a formula through conjunctions, disjunctions and negations.

b) Find a similar formula with only disjunctions and negations.

| $P_1$ | $P_2$ | $P_3$ | $f(P_1, P_2, P_3)$ |
|-------|-------|-------|--------------------|
| T     | T     | T     | T                  |
| T     | T     | F     | F                  |
| T     | F     | T     | F                  |
| T     | F     | F     | T                  |
| F     | T     | T     | F                  |
| F     | T     | F     | T                  |
| F     | F     | T     | T                  |
| F     | F     | F     | F                  |

$$A \wedge B = \sim(\sim A \vee \sim B)$$

$$\begin{aligned}
 f &\equiv (P_1 \wedge P_2 \wedge P_3) \vee \cancel{(P_1 \wedge P_2 \wedge \sim P_3)} \vee \cancel{(P_1 \wedge \sim P_2 \wedge P_3)} \vee \cancel{(P_1 \wedge \sim P_2 \wedge \sim P_3)} \\
 &\quad \vee \cancel{(\sim P_1 \wedge P_2 \wedge P_3)} \\
 &\equiv \sim(\sim P_1 \vee \sim P_2 \vee \sim P_3) \vee \sim(\sim P_1 \vee P_2 \vee P_3) \vee \sim(P_1 \vee \sim P_2 \vee P_3) \vee \\
 &\quad \sim(P_1 \vee P_2 \vee \sim P_3)
 \end{aligned}$$