Lecture 6.

Conditionals and Proofs (Sec. 1.2, 1.4)

1. Quiz 2.

Absorption Laws

(1) ~~[illegible]~~ $P \vee (P \wedge Q) \equiv P$;

(2) $P \wedge (P \vee Q) \equiv P$;

(3) $P \vee (\sim P \wedge Q) \equiv P \vee Q$;

(4) $\sim P \vee (P \wedge Q) \equiv \sim P \vee Q$;

(5) $P \wedge (\sim P \vee Q) \equiv P \wedge Q$

(6) $\sim P \wedge (P \vee Q) \equiv \sim P \wedge Q$.

[We can $A \equiv B$ say $A \Leftrightarrow B$ is T.

HA 3.

Extra Problems.

1. Sheffer's operation

$$P \square Q \equiv \sim P \wedge \sim Q \equiv \sim (P \vee Q);$$

$P \square Q \equiv \cancel{P} \; Q \square P$ - commutative since $\wedge$ is commutative

Associativity ?

$$P \square (Q \square R) \overset{?}{\equiv} (P \square Q) \square R$$

$$\equiv$$

$$\sim P \wedge \sim (\sim Q \wedge \sim R) \qquad \equiv$$

$$\equiv$$

$$\sim P \wedge (Q \vee R) \quad \not\equiv \quad \sim R \wedge (P \vee Q)$$

Sufficient to show that it's wrong for one combination True values (P, Q, R).

Take (F, T, T). Then on left - T

~~on~~ right - F.

2b)

$$P \wedge Q \qquad\qquad xy$$

$$\sim P \qquad\qquad x+1$$

$$P \vee Q \equiv \sim(\sim P \wedge \sim Q) \qquad\qquad xy + x + y$$

$$P \Rightarrow Q \equiv \sim P \vee Q \qquad\qquad (x+1)y + x+1+y \equiv \boxed{xy+}$$

2a)

$$y+z \qquad\qquad (\sim Q \wedge R) \vee (Q \wedge \sim R)$$

$$x(y+z) \qquad\qquad P \wedge [(\sim Q \wedge R) \vee (Q \wedge \sim R) =$$

$$= (P \wedge \sim Q \wedge R) \vee (P \wedge Q \wedge \sim R).$$

3)

$$P \vee Q \equiv (P \Rightarrow Q) \Rightarrow Q$$

$$\sim(P \Rightarrow Q) \equiv \sim(\sim P \vee Q) = P \wedge \sim Q$$

$$\cancel{P \wedge Q}$$

$$(P \Rightarrow Q) \Rightarrow Q \equiv \sim(P \Rightarrow Q) \vee Q$$

$$\equiv (P \wedge \sim Q) \vee Q \equiv P \vee Q$$

Peter – Math ⇒ Serge – ~Phys. ⇒ Serge is Chem.

But then Serge – ~Math. ⇒ Roma – Chem.

Contradiction!

Peter – ~Math ⇒ $\boxed{\text{Roman – Phys}}$ ⇒ $\boxed{\text{Peter – Chem.}}$

⇒ $\boxed{\text{Serge – Math.}}$

Examples of arithmetical prove:

We know the ~~modest~~ basic method of proofs - modus ponens:

Prove $P \Rightarrow Q$ we need show that if ~~of~~ P is True then Q is True.
When we verify that Q is True we use not only that P is True but some ~~pr~~ propositions about which we know that they are True:

- either we prove it earlier;
- either we consider them as evident.

The process of a construction of a mathematical theory is

the extension of set of True Theorem by proving new ones.

Proof of $P \Rightarrow Q$ can be a comlicate process from many steps using properties of conditio

We can not directly to show that Q is True but find an intermediate P_1 such that $P \Rightarrow P_1$ is true True Theorem and then P_1 is True. If $P_1 \Rightarrow Q$ is True then Q is True.

It corresponds to Tautology

$$((P \Rightarrow P_1) \wedge (P_1 \Rightarrow Q)) \Rightarrow (P \Rightarrow Q)$$

Ex. The number of intermediate P_j can be big: $((P \Rightarrow P_1) \wedge (P_1 \Rightarrow P_2) \wedge (P_2 \Rightarrow Q)) \Rightarrow (P \Rightarrow Q)$.

Let us look the Theorem on diagonals at rectangulars.

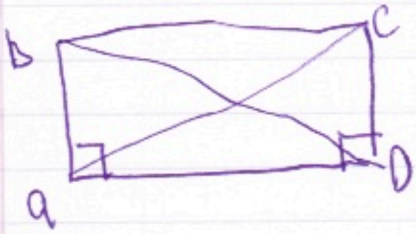
~~Byoo~~ Biconditionals correspond to Theorems with necessary and sufficient conditions.

Examples.

Theorem. A $\overbrace{\text{parallelogram}}^{P}$ is a rectangular iff its $\underbrace{\text{diagonals}}_{Q}$ are equal.

$P \Leftrightarrow Q$.

Proof. 1) $P \Rightarrow Q$. Let P is true and we have a restangular $ABCD$.



We want show ~~that~~ that the diagonals $AC = BD$.

Let us consider $\triangle BAD$ and $\triangle ADC$.

1. ~~Pr~~. $P \Rightarrow R$, R – "$\triangle BAD$ and $\triangle ADC$ are right (Angles A and D correspongdingly are right) on the definition of rectangular.

2. True Theorem $S \Rightarrow M$: Opposite sides at a parallelogram are equal. So $AB = CD$

3. True Theorem:

~~Oth~~ If corresponding legs at a right triangle are equal then the triangles are equal (congruent).

So $\Delta aBD = \Delta aDC$ since $aB = CD$ a the leg aB is joint.

So hypotenuses are equal $aC = BD$.

~~a~~ 2) $Q \Rightarrow P$.

Shortly! 1. $\Delta aBD = \Delta aCD$ (on 3 equal sides)

2. $\angle a = \angle D$

3. at parallelograms
$\angle a + \angle D = 180°$ (Theorem on parallel lines intersect. by a lin)

4. $\angle a = \angle D = 90°$.
Also $\angle C = \angle a = 90°$ (at parallelograms)
$\angle B = \angle D = 90°$

5. $\square aBCD$ is a rectangular $\boxtimes$

Arithmetic background.

Background. $\mathbb{N}$ - natural numbers + $\{0\}$.
Basic definitions & "evident" propositions:

$m | n$ (n is multiply of m) iff

$$n = qm \quad (q \text{ - quotient})$$

Proposition. For any (m, n) there are such (q, r) that

$$n = qm + r, \quad 0 \le r < m.$$

q - divisor
r - remainder.

They are unique.

Def. $n \neq 1$ is prime if n has no divisor different of 1 and n.

Proposition. Each n has an unique decomposition on prime factors

$$n = p_1 p_2 \cdots p_k.$$

Def. n is even if $2 | n$
n is odd

Simple Theorems.

1. $a|b \wedge b|c \Rightarrow a|c$

$$b = aq_1 \wedge c = bq_2 \Rightarrow c = aq_1q_2 \Rightarrow a|c.$$

2. Sum of odd numbers is even and product is odd

$$a = 2m+1 \wedge b = 2n+1 \Rightarrow a+b = 2(m+n+1)$$
$$a \cdot b = 2(2mn+m+n)+1$$

3. Squares of natural number m^2, n^2.
Then m^2-n^2 is odd or $4 | m^2-n^2$.

Hint

Let $m \geq n$. $m^2-n^2 = (m+n)(m-n)$. Factors either both even, or both odd.