Lecture 5.

Conditionals, Biconditionals, Theorems.

(Sect. 1.2, 1.4).

Most of Theorems have ~~of~~ the form of conditionals $P \Rightarrow Q$.

at usual texts conditionals can be True under some circumstances and False under another ones but

True Teorems are True always - (they are Tautologies!).

How do we prove Theorems $P \Rightarrow Q$?

We ~~ask~~ show that each time when P is True, then Q is also True.

$P \Rightarrow Q$
P - antecedent
Q - consequent.

How do we apply (True) Theorems?

If $P \Rightarrow Q$ is True Theorem and ~~Q~~ P is True then Q is also True.
(modus ponens).

Tautology ~~$(P \wedge Q)$~~
$$((P \Rightarrow Q) \wedge P) \Rightarrow Q$$

But if $P \Rightarrow Q$ and Q are True we can't conclude that ~~Q~~ P is True.

We considered several examples. One more situation.

Let $A(x) = B(x)$ is an algebraic equation (P

Then $A^2(x) = B^2(x)$ (Q

$P \Rightarrow Q$ is True (other words: each root x of P is a root of Q).

But conversion $Q \Rightarrow P$ can be False (but sometimes is True!).

Ex. $x - 2 = \sqrt{-2x+7}$ (1)

$(x-2)^2 = -2x + 7$ (2)

$x^2 - 2x - 3 = 0$ Roots $x_1 = -1$, $x_2 = 3$

~~But~~ $x_2 = 3$ is a root of (1) (extraneous root)

But $x_1 = -1$ isn't

In the opposite direction $(Q \Rightarrow P)$ roots can be lost.

So for True Theorem $P \Rightarrow Q$, the converse Theorem can be False.

Biconditional connective $P \Leftrightarrow Q$

corresponds P iff Q, P is equivalent
P is necessary and sufficient for Q

P	Q	$P \Leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

$$P \Leftrightarrow Q \equiv (P \Rightarrow Q) \wedge (Q \Rightarrow P).$$

Connectives $f(P_1, \dots, P_n)$ and $g(P_1, \dots, P_n)$ are equivalent iff
$f(P_1, \dots, P_n) \Leftrightarrow g(P_1, \dots, P_n)$ is Tautology.

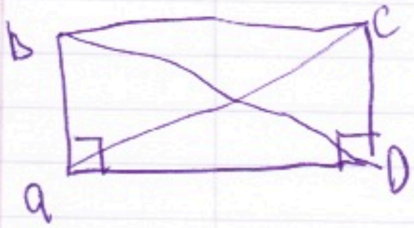
Biconditionals correspond to Theorems with necessary and sufficient conditions.

Examples.

Theorem. A $\overbrace{\text{parallelogram}}^{P}$ is a rectangular iff its $\underbrace{\text{diagonals}}_{Q}$ are equal.

$P \Leftrightarrow Q$.

Proof. 1) $P \Rightarrow Q$. Let P is true and we have a restangular $ABCD$.



We want show ~~that~~ that the diagonals

$AC = BD$.

Let us consider $\triangle BAD$ and $\triangle ADC$.

1. ~~Prn.~~ $P \Rightarrow R$, R – "$\triangle BAD$ and $\triangle ADC$ are right (Angles A and D corresponqdingly are right)" on the definition of rectangular.

2. True Theorem $S \Rightarrow M$: Opposite sides at a parallelogram are equal. So $AB = CD$

... the Theorem:

~~...~~ If corresponding legs at a right triangle are equal then the triangles are equal (congruent).

So $\Delta aBD = \Delta aDC$ since $aB = CD$ a the leg aB is joint.

So hypotenuses are equal $aC = BD$.

2) $Q \Rightarrow P$.

Shortly! 1. $\Delta aBD = \Delta aCD$ (on 3 equal sides)

2. $\angle a = \angle D$

3. at parallelograms

$\angle a + \angle D = 180°$

4. $\angle a = \angle D = 90°$.

Also $\angle C = \angle a = 90°$ (at parallelograms)

$\angle B = \angle D = 90°$

5. $\square aBCD$ is a rectangular $\boxtimes$

We'll try to understand how to prove Theorems

- analyse specific methematical proofs (1)
- analyse logical tools (2)
- solve problems on proofs (3)

Properties of Convolution can be ~~a~~ usefull

~~In proper~~

Propositions 2.1 Next connectives are equivalent to Convolution $P \Rightarrow Q$

(i) $\sim Q \Rightarrow \sim P$; (contraposition)

(ii) $(P \wedge \sim Q) \Rightarrow \sim P$;

(iii) $(P \wedge \sim Q) \Rightarrow Q$;

(iv) $(P \wedge \sim Q) \Rightarrow F$

Contradiction (any identical False), i.e. F

- How to prove these equivalencies?

- How to use them?

We define $P \Rightarrow Q$ by its True Table:

P	Q	$P \Rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

The table follows to the principle « From Truth follows ~~of~~ only Truth but from Lie can follow both Truth and Lie. »

Proporties:

1. $P \Rightarrow Q \equiv \sim P \vee Q$
2. $\sim (P \Rightarrow Q) \equiv P \wedge \sim Q$
3. $P \Rightarrow (Q \Rightarrow R) \equiv (P \wedge Q) \Rightarrow R$
4. $P \Rightarrow (Q \wedge R) \equiv (P \Rightarrow Q) \wedge (P \wedge R)$
5. $(P \vee Q) \Rightarrow R \equiv (P \Rightarrow R) \wedge (Q \Rightarrow R)$.

True tables or narratives.