Lecture 4.

1. Quiz 1.

2. Selective problems from HA 2.

Extra problems:

1. Alternative (or exclusive) disjunction

"Either P or Q"

True Table

P	Q	$f(P, Q)$
T	T	F
T	F	T
F	T	T
F	F	F

$$f(P,Q) \equiv (\sim P \wedge Q) \vee (P \wedge \sim Q).$$

2. Majority connective

$$f(P_1, P_2, P_3, P_4) \equiv (P_1 \wedge P_2 \wedge P_3 \wedge P_4) \vee (\sim P_1 \wedge P_2 \wedge P_3 \wedge P_4) \vee$$
$$(P_1 \wedge \sim P_2 \wedge P_3 \wedge P_4) \vee (P_1 \wedge P_2 \wedge \sim P_3 \wedge P_4)$$
$$\vee (P_1 \wedge P_2 \wedge P_3 \wedge \sim P_4).$$

3. We have

$$P \wedge Q \equiv \sim(\sim P \vee \sim Q) \ ; \ P \vee Q \equiv \sim(\sim P \wedge \sim Q).$$

$$(\sim P \vee Q) \wedge \sim (\sim R \vee \sim S) \equiv$$

$$\sim\big(\sim(\sim P \vee Q) \vee (\sim R \vee \sim S)\big) ; \text{ (no } \wedge)$$

$$\equiv \sim(P \wedge \sim Q) \wedge (R \wedge S) \quad \text{(no } \vee)$$

4*. No! If $f(P_1, \dots, P_2)$ is a composition of conjunctions and disjunctions then $f(T, T, \dots, T) = T$ and we can't represent $\sim P$.

5*. If $\underbrace{3 \nmid n}_{P}$ then $\underbrace{\text{the remainder of division } n^2 \text{ on } 3 \text{ is equal } 1.}_{Q}$

$P \Rightarrow Q$.

1. Transform the narrative at formula

$P: 3 \nmid n$

Q: remainder of n is 1 or 2:

$n = 3k+1$ or $n = 3k+2$.

2. $3 \nmid n \Rightarrow n^2 = 9k^2 + 6k + 1 = 3m + 1$ ✓

$n^2 = 9k^2 + 12k + 4 = 3m + 1$ $\boxed{}$

Remark. If $n = 3k+2$ then also $n = 3l - 1$.

5.* 1. One of Andre or Dan lied.

2. Roger also lied since if he said Truth then 3 lied (Nick, Alex and {Andre or Dan}).

3. So lied Roger and one of {Andre or Dan}. Nick and Alex said ~~Truth~~ Truth.

4. Dan said Truth, Andre lied

Lie	Truth
Andre, Roger	Nick, Alex, Dan

5. Roger left flowers.

~~Ok~~ So connectives corresponds to polynomials mod 2.

Conditional connective (Sect. 1.2) (or implication)

$P \Rightarrow Q$

corresponds many ~~sign~~ grammatical constructions:

"if P then Q"

"from P follows Q"

"Q is necessary condition of P".

Ex. P - "A number is multiple of 6"

Q - " 3".

$P \Rightarrow Q$ is a True conditional

Again we are interesting just True values but don't consider any causal connections between P and Q. and it ~~al~~ often looks ambigouss.

We define $P \Rightarrow Q$ by its True Table:

P	Q	$P \Rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

The table follows to the principle « From Truth follows only Truth but from Lie can follow both Truth and Lie. »

Properties:

1. $P \Rightarrow Q \equiv \sim P \vee Q$
2. $\sim (P \Rightarrow Q) \equiv P \wedge \sim Q$
3. $P \Rightarrow (Q \Rightarrow R) \equiv (P \wedge Q) \Rightarrow R$
4. $P \Rightarrow (Q \wedge R) \equiv (P \Rightarrow Q) \wedge (P \wedge R)$
5. $(P \vee Q) \Rightarrow R \equiv (P \Rightarrow R) \wedge (Q \Rightarrow R)$.

True tables or narrative.

So prove that Theorem $P \Rightarrow Q$ is True we need

prove that if P is True then Q is also True.

~~Problem~~

Properties can help.

6. $P \Rightarrow Q \equiv \sim Q \Rightarrow \sim P$ (contraposition)

~~True~~ False iff $P = T, Q = F$	False iff $\sim Q = T, \sim P = F$

The same.

It's the base of proofs by contradiction.

Warning! Big mistake!

$P \Rightarrow Q$; Suggest that Q - True, make a True conclusion and decide that P is also True.

P – " $6 \mid 15$ "

Q – " $3 \mid 15$ "

$P \Rightarrow Q$

Q is Truth but Tru

P is False, $P \Rightarrow Q$ is ~~False~~

[Bushism:

Iraq's war was positive since it destoyed Saddam Hussein but he was criminal.]

Informaly: ~~Good~~ Bad things can give good results!

$P \Rightarrow Q$. <u>Conversion:</u> $Q \Rightarrow P$.

Inverse Theorem.

Theorem inversed to a true Theorem can be false.

P - "a ~~n~~ number is multiple too 15"

Q " — " — 5"

$P \Rightarrow Q$ is True, but $Q \Rightarrow P$ can be False (Theorem, which is inverse to a True Theorem, can be False one).