Lecture 25.

1. MT 2 (solutions)

2. Direct computations of limits.

Plan $\lim_{n\to\infty} x_n = L$

$$(\forall \varepsilon)\ \varepsilon>0 \Rightarrow \{(\exists N)(\forall n)(n>N \Rightarrow |L-x_n|<\varepsilon)\}$$

1) Guess: $L=?$

2) Analysis: choice $N(\varepsilon)$.

3) Proof.

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1. $\lim_{n\to\infty} \frac{n-1}{n} = ?$

1) Guess.

$L=1$.

2) Analysis

$|1-x_n| = \frac{1}{n} \Rightarrow N(\varepsilon)$ can be taken as $N(\varepsilon) = [\frac{1}{\varepsilon}]$.

for $x_n = \frac{1}{n}$.

3) $n > [\frac{1}{\varepsilon}] \Rightarrow n > \frac{1}{\varepsilon} \Rightarrow \frac{1}{n} = |1-x_n| < \varepsilon$. True.

2. $x_n = \frac{1}{n^2}$.

1) $\lim = 0$.

2) $|x_n| = \frac{1}{n^2} \leq \frac{1}{n}$. So again we can take $N(\varepsilon) = [\frac{1}{\varepsilon}]$

3) $n > N(\varepsilon) = [\frac{1}{\varepsilon}] \Rightarrow$ ~~|1-x_n|~~ $|0 - x_n| = \frac{1}{n^2} < \frac{1}{n}$

$< \frac{1}{[\frac{1}{\varepsilon}]} < \frac{1}{\frac{1}{\varepsilon}} = \varepsilon$.

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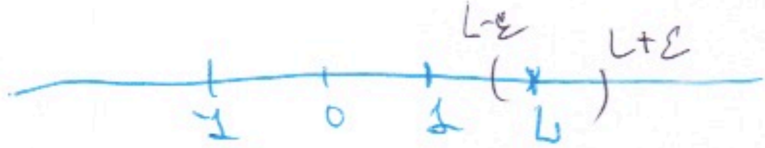
3. $x_n = (-1)^n$: $-1, 1, -1, \ldots$

1) Guess: No limit! (disverges)



$(\forall L)(\exists \varepsilon)\ \varepsilon > 0 \wedge \{(\forall N)(\exists n)[n > N \wedge |L - x_n| \geq \varepsilon]\}$

$\boxed{\varepsilon(L)}$ Take such $\varepsilon = \varepsilon(L) > 0$ s.t.

$\boxed{|L-1| > \varepsilon \vee |L+1| > \varepsilon}$ (*)

Then a "half" of x_n "will $|x_n - L| > \varepsilon$

Let $|L-1| > \varepsilon$.

So 1 is outside of ε-neighborhood of L.
The case of $|L+1|>\varepsilon$ is considered similarly.

Remark: We can ~~ca~~ take ~~$\varepsilon\Rightarrow$~~ $\varepsilon(L)=\frac{1}{2}$ (independent of L!). Then $x=-1, 1$ can't be both ~~lie~~ simultaneously ~~in~~ / ~~at~~ $|L-x|<\varepsilon$.

Why?

If ~~$|L+1|$~~ $|L-1|>\varepsilon$ for any $(\forall N)$ we can take any ~~nod~~ even n and then $x_n=1 \wedge |L-x_n|>\varepsilon$.

$$\lim_{n\to\infty}\frac{1}{2^n}=L\ ?$$

1) $L=0$

2) $|L-x_n|=\frac{1}{2^n}<\varepsilon \Leftrightarrow \frac{1}{\varepsilon}<2^n$

$$N(\varepsilon)=\left[\log_2\frac{1}{\varepsilon}\right]=[-\log_2\varepsilon].$$