Lecture 23.

Review to MT2.

Predicatives, axiomatics.

{1.3, 1.1)

1. Predicative algebra.

- Predicatives; subjective fields (sets) $\mathbb{N}, \mathbb{R}$ ...

$a(x)$, $P(x,y)$

2. Quantifiers

$(\forall x)$ universal

$(\exists x)$

$\land (\exists z) a(z)$

$(\exists ! x)\, a(x) \Leftrightarrow (\exists x)\, a(x) \land \big((\exists y)\, a(y) \Rightarrow y = z\big)$.

3. Properties

~~a~~ - interchange

- convenient negation

$\sim (\forall x)\, a(x) \Leftrightarrow (\exists x) \sim a(x)$

$\sim (\exists y)\, B(x) \Leftrightarrow (\forall x) \sim B(x)$

Important formulas

$$\sim(P\Rightarrow Q) \Leftrightarrow P\wedge \sim Q\ ; \quad \sim(P\Leftrightarrow Q) \Leftrightarrow (\sim P\wedge Q)\vee(P\wedge \sim Q).$$

$$\cancel{\sim(\exists! x)a(x) \Leftrightarrow \left((\forall x)\sim a(x) \vee (\exists y)a(y)\wedge y\neq x\right)}$$

$$\sim(\exists! x)a(x) \Leftrightarrow (\forall x)\sim a(x) \vee \left((\exists y)(\exists z)\ a(y)\wedge a(z)\wedge y\neq z\right)$$

Restrictide quantifiers:

$(\forall x)\ P(x) \Rightarrow$

$(\exists x)\ Q(x) \wedge$

Tautologies for quantifiers.

- - ~ .

Structures of axiomatic theories

- basic predicats
- axioms

- new predicat
- Theorem

Geometry

- Arithmetic

Sequences of real numbers

$$a_1, a_2, \ldots, a_n, \ldots$$

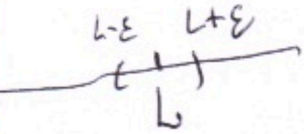
Ex. $\frac{1}{n}$; $\frac{(-1)^n}{n}$; $\frac{\sin n}{n}$; $3, 3, \ldots$, 2^n,

Principal definition:

$\mathbb{R}$ $\mathbb{N}$

$L = \lim_{n \to \infty} x_n$ iff

$$(\forall \varepsilon)\Big[\varepsilon > 0 \Rightarrow \big((\exists N)(\forall n)[n > N \Rightarrow |L - x_n| < \varepsilon]\big)\Big]$$



Negation

$L \neq \lim_{n \to \infty} x_n$ iff

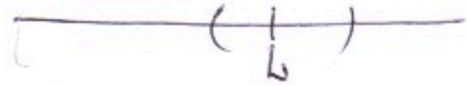
$$(\exists \varepsilon)\Big[\varepsilon > 0 \wedge \big((\forall N)(\exists n)[n > N \wedge |L - x_n| \geq \varepsilon]\big)\Big]$$

The sequence disverges iff

$(\forall L)$

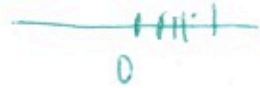
Remark:

$|x_n - L| < \varepsilon \Leftrightarrow L - \varepsilon < x_n < L + \varepsilon$



Examples.

1. $x_n = \frac{1}{n}$. Guess: $L = 0$



~~Prove~~ Proof: Take $(\forall \varepsilon)$ $\varepsilon > 0$

Analysis: $|x_n - 0| = \left|\frac{1}{n} - 0\right| = \frac{1}{n}$

When $\frac{1}{n} < \varepsilon$? If $\frac{1}{\varepsilon} < n$

So take $N(\varepsilon) = [\frac{1}{\varepsilon}]$

integer part.

Then $\frac{1}{\varepsilon} < n$ if $n > N(\varepsilon)$.

2. $x_n = \frac{1}{3n}$. Again $L = 0$.

$\varepsilon > 0$. ~~Whe~~ Which $N(\varepsilon)$ can we take?

$\frac{1}{3n} < \varepsilon \Leftrightarrow \frac{1}{3\varepsilon} < \varepsilon$. So $N(\varepsilon) = \left[\frac{1}{3 \cdot \varepsilon}\right]$

We can always take $N(\varepsilon)$ greater

3. $x_n \equiv 3$. $L = 3$

$N(\varepsilon) = 1$

$|x_n - L| = 0$

4. $x_n = \frac{1}{2^n}$.

$L = 0$ $|x_n - L| = \frac{1}{2^n} < \varepsilon \Leftrightarrow \frac{1}{\varepsilon} < 2^n$

$N(\varepsilon) = [\log_2 \frac{1}{\varepsilon}] \Rightarrow n > N(\varepsilon) \Rightarrow 2^n < \frac{1}{\varepsilon}$.

5. $x_n = \frac{n}{n+1}$.

Guess. $L = 1$

$|L - x_n| = \frac{1}{n+1}$.

So we can take $N(\varepsilon) = [\frac{1}{\varepsilon}] + 1$ as for

$x_n = \frac{1}{n+1}$

Remark. $\frac{n}{n+1} = 1 - \frac{1}{n+1}$

6. $x_n = \frac{1}{n^2}$. $\lim_{n \to \infty} \frac{1}{n^2} = 0$

Reason: $0 < \frac{1}{n^2} < \frac{1}{n}$. Why?