Lecture 20.

Method of Mathem. Induction.

HW 10.

#5 (a) $a_k = 5k$, $k \in \mathbb{N}$

Ind. def. (i) $a_1 = 5$

(ii) $a_{k+1} = a_k + 5$.

(b) $n \in \mathbb{N}$, $n > 10$

(i) $a_1 = 11$,

(ii) $a_{k+1} = a_k + 1$.

#6 g(d)

S_n $1 \cdot 1! + \dots + n \cdot n! = (n+1)! - 1$

(i) Base. $S_1 = 1$. True.

(ii) Inductive conditional $P \Rightarrow Q$

$S_k = (k+1)! - 1 \Rightarrow S_{k+1} = (k+2)! - 1$.

P $S_k \Rightarrow S_{k+1} = (k+1)! - 1 + (k+1)(k+1)! =$

$= (k+1)!\,(1 + k + 1) - 1 = (k+2)! - 1 \Leftrightarrow Q$.

6e) $S_n = 1^3 + 2^3 + \dots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$

(i) $S_1 = 1$. True

(ii) $S_k = \left[\frac{k(k+1)}{2}\right]^2 \Rightarrow S_{k+1} = \left[\frac{(k+1)(k+2)}{2}\right]^2$

$S_k = \left[\frac{k(k+1)}{2}\right]^2 = S_{k+1} = \cancel{\left[\frac{k(k+1)(k+2)}{2}\right.}$

$= \left[\frac{k(k+1)}{2}\right]^2 + (k+1)^3 = (k+1)^2\left[\frac{k^2+4(k+1)}{4}\right]$

$= \frac{(k+1)^2}{4} \cdot (k+2)^2$. Conditional is True.

6g) $S_n = \frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \dots + \frac{1}{n(n+1)} = \frac{n}{n+1}$

(i) $S_n = \frac{1}{2}$. True

(ii) $S_k = \frac{k}{k+1} \Rightarrow S_{k+1} = \frac{k}{k+1} + \frac{1}{(k+1)(k+2)} =$

$= \frac{1}{k+1}\left(k + \frac{1}{k+2}\right) = \frac{1}{k+1}\left(\frac{k^2+2k+1}{k+2}\right) =$

$= \frac{(k+1)^2}{(k+1)(k+2)} = \frac{k+1}{k+2} = S_{k+1}$. True

condition.

Remark. $\frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$.

So $S_n = \left(1 - \cancel{\frac{1}{2}}\right) + \left(\cancel{\frac{1}{2}} - \cancel{\frac{1}{3}}\right) + \dots \left(\cancel{\frac{1}{n}} - \frac{1}{n+1}\right) =$

$= 1 - \frac{1}{n+1} = \frac{n}{n+1}$

1a) $3 \mid n^3+5n+6$. $3 \mid a_n$.

(i) $n=1$. $3 \mid 12$. True

(ii) $3 \mid k^3+5k+6 \Rightarrow a_{k+1} =$

$= (k+1)^3 + 5(k+1) + 6 = k^3 + 5k + \cancel{3(k+1)^2 +}$

$\cancel{+3(k+1)} + 3k^2 + 3k + 1 + 5 + 6 =$

$= a_k + 3(k^2+k) + 12 \Rightarrow 3 \mid a_{k+1}$.

7c) $6 \mid a_n = n^3 - n$

(i) $n=1$. $6 \mid 0$. True

(ii) $a_{k+1} = (k+1)^3 - k - 1 = k^3 - k + 3(k^2+k)$

$= a_k + 3k(k+1)$. $2 \mid k(k+1)$.

$6 \mid a_k \Rightarrow 6 \mid a_{k+1}$.

Remark. $n^3 - n = (n-1)n(n+1)$ – product 3 consequent numbers $\Rightarrow 6 \mid n^3 - n$.

7d) $12 \mid (n^3-n)(n+2) = a_n$

Hint. Prove that $12 \mid a_{k+1} - a_k$.

Remark $a_n = (n-1)n(n+1)(n+2)$ – product of 4 consequantive numbers.

In reality $24 \mid (n^3-n)(n+2)$.

7g) $8 \mid 9^n - 1$.

(i) $n=1$. $8 \mid 8$. True

(ii) ~~$9^{k+1} - 1 = (9^{k+1} - 9^{k}) + (9^{k} - 1) = 8 \cdot 9^k + 8$~~

~~$8 \mid 9^k \Rightarrow$~~

$9^{k+1} - 1 = (9^{k+1} - 9^k) + (9^k - 1) = 8 \cdot 9^k + (9^k - 1)$.

$8 \mid 9^k - 1 \Rightarrow 8 \mid 9^{k+1} - 1$.

7j) $4^{n+4} > (n+4)^4$

(i) $n=1$. $4^5 > 5^4$.

$\left(4^{k+4} > (k+4)^4\right) \overset{?}{\Rightarrow} 4^{k+5} > (k+5)^4$

We need prove that

$$4 > \left(\frac{k+5}{k+4}\right)^4 \Leftrightarrow 4 > \left(1 + \frac{1}{k+4}\right)^4 \Leftrightarrow$$

$$2 > \left(1 + \frac{1}{k+4}\right)^2;$$

$$1 + \frac{1}{k+4} < \frac{6}{5} \quad \vee \quad 2 > \frac{36}{25}$$

Remark. Convenient put $m = n+4$.

$m \geq 5$.