Lecture 2.

Propositional Algebra (continuation)

(1) 2 problems from HA 1.

You need to think not only about a solving but about the presentation of solution.

1. Prove that if an even number is a square then it is multiple of 4.

Proof. 1) Transform the narrative at formulas.

$\mathbb{N}$ natural numbers

Data: Let $n \in \mathbb{N}$, $2 | n$, $n = k^2 \Rightarrow$ 2) k is even (since products of odd numbers are odd).

3) "k is even" means $k = 2l$.

4) Then $k^2 = 4 l^2$ and $4 | k^2 = n$. $\blacksquare$

2. "Problem about Alice":

The question must be a connective ~~of~~ (propositional function)

$$f(P, Q)$$

of 2 statements

P - We are at city A.

Q - You say me Truth.

Let us construct True Table for $f(P, Q)$:

P	Q	$f(P, Q)$
T	T	T
T	F	F
F	T	F
F	F	T

- If Q is True (1 and 3th lines) we want that value of f would coincide with P.

- If Q is Lie the value of f must be opposite to values of P (2 and 4th lines)

Why?

We can take as $f(P, Q)$ the connective

$$\left(P \wedge Q\right) \vee \left((\sim P) \wedge (\sim Q)\right)$$

which has this True Table.

[Is it true that
we are at the city A and
you say Truth to me?]
or we are at the city B and
you lied?]

Samples of problems.

1. Tautology — Propositional formula which True ~~valye~~ value is always Truth.

Ex. $P \vee (\sim P) \equiv T$

Contradiction – Propositional formula which True value is always False.

Ex. $P \wedge (\sim P) \equiv F$

T, F are notations for Tautologies and Contradictions.

They play a role of "constants" T, F.

How to prove: True tables or transformations using Laws 1-9

Problems.

1. ~~Simplify~~ Equivalent transformation

$$((\sim P) \wedge Q) \vee (\sim (R \vee (\sim S))) \equiv$$

$$\equiv ((\sim P) \wedge Q) \vee ((\sim R) \wedge S) \quad \text{De Morgan; double negation}$$

$$\equiv (((\sim P) \wedge Q) \vee (\sim R)) \wedge (((\sim P) \wedge Q) \vee S) \quad \text{distributi[ve]}$$

$$\equiv (\sim P \vee \sim R) \wedge (Q \vee \sim R) \wedge (\sim P \vee S) \wedge (Q \vee S) \quad \text{distrib[utive] \& associativi[ty]}$$

Useful equivalences:

$(P \wedge P) \equiv P$

$(P \vee P) \equiv P$

$P \wedge (P \vee Q) \equiv P$

$P \vee (P \wedge Q) \equiv P.$

$P \wedge (\sim P \vee Q) \equiv P \wedge Q$

$P \vee (\sim P \wedge Q) \equiv P \vee Q$

$(P \wedge \sim P) \vee (P \wedge Q) \equiv$
$F \vee (P \wedge Q) \equiv P \wedge Q$

$P \wedge \sim P \equiv F$, $F \vee R \equiv R$

$P \vee \sim P \equiv T$, $T \wedge R \equiv R.$

Problem. John went to the class and learnt that Liz didn't went to the library or the stadium.

Choose elementary propositions and write down this sentence as propositional formula.

P - John went to the class;

Q - Lis went to the library

R - ——————" stadium.

$P \wedge \sim(Q \vee R) \equiv$

$P \wedge \sim Q \wedge \sim R$.

Construct negation!

$\sim P \vee Q \vee R$

John didn't be in the class or

~~Liz~~ went at library or stadium.

Pay attention on assotiativity!

Problem. How many non equivalent connectives $f(P,Q)$ exists.

answer. ~~$\emptyset$~~ $16 = 2^4$,

Since there are 4 lines at True table and at which $f(P,Q)$ ~~it~~ takes T or F.

<u>Proposition</u>, There are $2^{(2^n)}$ non equivalent connectives $f(P_1, \dots, P_n)$ of n variables.

Proof. The are 2^n lines $(\sigma_1, \dots, \sigma_n)$ at True tables and 2 possibilities T, F for True values of f at these lines (the last column).

Problem. Can we present each connective $f(P_1, \dots P_n)$ by a formula through $\sim, \vee, \wedge$ (up to equivalency)?

Answer. Yes.