Lecture 17.

Extra Problem. As at the problem about sages each of alchemist tries to imagine what think other ones after the order. Some of them know 7 ~~an~~ dishonest servants, some 6.

What do think the last ones?

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What is clear?

If ~~somebody~~ somebogy didn't know any traitor then his servent is traitor and he would be kill him;

If nobody was killed ~~f~~ at the 1st night on 2nd night ~~other~~ alchemists who know only 1 traitor will ~~a~~ kill his one etc.

all 7 traitor will be killed at 7th night by ~~the~~ ~~oth~~ alchemists who knows only 6 traitors

the number of alchemists isn't essential.

3) $a = (\forall x)(P(x) \vee Q(x))$

$b = (\forall y)P(y) \vee \forall z\, Q(z))$

$B \Rightarrow a$ - Tautology:

$(\forall y)P(y)$ - T means $P(y) \equiv T$

B means at least one of them is Tautology but their disjoncition is True

But if a is True it means that for each x $P(x)$ or $Q(x)$ is True but it doesn't mean that ~~one of the~~ at least of them is tautology.

Typical example:

$[\mathbb{N}]$ $P(x)$: x is odd; $Q(x)$: x is ~~odd~~ even

Then $(\forall x)\ P(x) \vee Q(x)$ is True

but $(\forall y)P(y)$ and $(\forall z)Q(z)$ are False.

Axiomatic theories

1. Fix subject fields
2. Denote and give names basic predicats.
3. New predicats are ~~defined~~ defined through basic ones
4. Some closed formulas are taken as True one (Axioms or Postulates)
5. Other theorems are ~~the~~ consequences of axioms.

First example – Euclidian ~~geom~~ geometry on the plane. (IV cent. BC).

Basic subject fields
- Points ($\mathcal{P}$)
- Lines ($\mathcal{L}$)

Basic predicats:

1. $x \in l$ (x - point, l - line),
2. $x < y < z$ (point y lies on the line l between x, z)

More exact: only if $(\exists l)\ x \in l \wedge y \in l \wedge z \in l$

New predicats

3. Interval $z\in(x,y) \Leftrightarrow$ ~~$(\forall z)$~~ $x<z<y$.

4. x is the intersection point of l, m

$$x\in l \wedge x\in m$$

5. $l \neq m$ $\nparallel$

6. $l \parallel m$

7. tringle Δxyz

$(x,y)\equiv(u,v)$ – congruency

Axioms

1. $(\forall l)(\exists x)(\exists y)\ x\in l \wedge y\in l \wedge x\neq y$

(on each line there are at least 2 different points)

2. $(\forall l)(\exists x) \sim x\in l$

(for each line there is a point outside the line

3. $(\forall x)(\forall y)(\forall u)(\forall v)(\forall t)(\forall s)\ ((x,y)\equiv(u,v)\wedge(u,v)\equiv(s,t)$

(transitivity of congruency)

3'. ~~$(\forall x)(\forall x)$~~ $(\forall x)(\forall y)(\forall z)(\forall l)(x\in l\wedge y\in l \wedge x<z<l) \Rightarrow z\in l$.

$(\forall x)(\forall y)(\forall l)\ x\in l \wedge y\in l \Rightarrow (\exists z)\ z\in l \wedge \sim x<z<y$.

4. $(\forall x)(\forall y)\ x \neq y \Rightarrow (\exists! l)\ x \in l \wedge y \in l$

~~There is a line~~

~~Through~~ For 2 different ~~Pla~~ points ~~a~~ there is a line passing through these points and this line is unique.

Theorem. Two different lines ~~can~~ intersect not more than 1 point.

$$(\forall l)(\forall m)\ l \neq m \Rightarrow \big((\forall x)(\forall y)(x \in l \wedge x \in l \wedge y \in m \wedge x \in m) \Rightarrow x = y\big)\big)$$

Proof. By contradiction. $\sim Q \Rightarrow (\exists x)(\exists y)\ x \in l \wedge x \in m \wedge y \in l \wedge y \in m \wedge l \neq m$

Definition ~~*~~ (Intersection point). Contradiction with axiom 4.

$$x = l \cap m = x \in l \wedge x \in m$$

Proof. Contradiction. ~~∃Q~~

$\sim Q:\ (\exists x)(\exists y)\ x \in l \wedge y \in l \wedge x \in m \wedge y \in m \wedge x \neq y$

axiom: $(\exists! \cancel{\ell}\ l)\ x \in l \wedge y \in l$

Contradiction: $l \neq m$

5th postulat. (axiom) $(\forall l)(\forall x) \sim x \in l \Rightarrow (\exists m)\ \cancel{\ } x \in m \wedge m \parallel l$

~~$E(a,b)$~~, $l(x,y)$, $x \neq y$ "line l is the unique line passing through x, y.

There are several axiom for each basic predicats:

"lie between":

$$(\forall x)(\forall y)(\forall z)\, x < y < z \Rightarrow \sim (x < z < z \wedge y < x < z).$$

~~Corollary:~~

$(\forall x)(\forall y)(\forall z)(\forall l)\,(x, y, z) \in l \Rightarrow (\exists!)$ ~~$x \vee y \vee z$ such~~ $\{x, y, z\}$ which lies between 2 other ones.

It's possible write by a formula

$$(\forall x)(\forall y)(\forall l)\; x \in l \wedge y < l \Rightarrow (\exists z)(\exists u)\; x < z < y \wedge$$
$$\wedge\, x < y < u.$$

Corrolary: There is ∞ set of points on any line.

4. $(\forall x)(\forall y)\ x \neq y \Rightarrow (\exists!\, l)\ x \in l \wedge y \in l$

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Definition (Intersection point)

$$x = l \cap m = x \in l \wedge x \in m$$

Contradiction with axiom 4.

Proof. Contradiction.

$\sim Q$: $(\exists x)(\exists y)\ x \in l \wedge y \in l \wedge x \in m \wedge y \in m \wedge x \neq y$

axiom: $(\exists! \, l)\ x \in l \wedge y \in l$

Contradiction: $l \neq m$

5th postulat. (axiom) $(\forall l)(\forall x) \sim x \in l \Rightarrow (\exists m)\ x \in m \wedge m \parallel l$