~~So # pages = 318 − 182 = 136~~

Lecture 13.

Properties of quantifiers.

1. Unique existential quantifier

$(\exists! x)\, P(x)$ {exist a unique x s.t. $P(x)$ is True.

$$(\exists! x)\, P(x) \Leftrightarrow (\exists x)\, P(x) \wedge (\forall y)(\forall z)\, P(y) \wedge P(z) \Rightarrow y = z.$$

Ex. $(\forall x)(\exists! y)\ y^2 = x$ $(\mathbb{R}_+)$

$(\forall y)(\exists! x)\ y = 2^x$ $[x : \mathbb{R};\ Y : \mathbb{R}_+]$.

2. Using brackets always show ~~an~~ areas of actions of quantifiers.

Ex. $\forall x\, (P(x, y) \vee (\exists y)(Q(y)))$.

$\forall x\, (x = 0 \vee (\exists y)\ y = \frac{1}{x})$ $(\mathbb{R})$.

$\forall a(x) \Leftrightarrow (\forall y)\, a(y) \wedge (\forall z)\, a(z))$

3. $\forall$ is „a relative" of the conjunction $\wedge$
$\exists$ — " — disjunction $\vee$

Let the subject field is finite:
$X = \{x_1, \dots, x_n\}$.

Then
$$(\forall x) P(x) \Leftrightarrow P(x_1) \wedge \dots \wedge P(x_n)$$
$$(\exists x) P(x) \Leftrightarrow P(x_1) \vee \dots \vee P(x_n).$$

4. Similar ~~&~~ quantifiers can be interchange:
$$(\forall x)(\forall y) P(x,y) \Leftrightarrow (\forall y)(\forall x) P(x,y)$$
$$(\exists x)(\exists y) P(x,y) \Leftrightarrow \cancel{(\exists y)(\exists x) P(x,y)}$$
$$(\exists y)(\exists x) P(x,y).$$

But non similar — not

$$(\forall x)(\exists y) P(x,y) \not\Leftrightarrow (\exists y)(\forall x) P(x,y)$$

$P(x,y)$: Student x takes course y.

$\forall x\, \exists y\, P(x,y)$: Each student takes some course. (y depends of x) $y(x)$

$\exists y\, \forall x$: Some ~~student~~ course takes all ~~courses~~ students

Remark: $\exists y \forall x\, P(x,y) \Rightarrow \forall x\, \exists y\, P(x,y)$ is True!

$\Leftarrow$ not.

$X = \mathbb{R}_+$ $\quad \cancel{\forall x\, \exists}$

$\forall y\, \exists x\;\; y = x^2$ is true

but $\exists x\, \forall y\;\; y = x^2$ is False.

4. Negation of quantifiers.

$$\sim (\forall x)\, P(x) \Leftrightarrow (\exists x)\,(\sim P(x))$$
$$\sim (\exists x)\, P(x) \Leftrightarrow (\forall x)\,(\sim P(x))$$

(not all cats are gray $\overset{?}{\Leftrightarrow}$ all cats are not gray)

Ex. Not all polynomials have real roots

$\Updownarrow$

There is a polynomial without real roots.

It' similar to convenient formulas for propositional connectives