Lecture 11.

Review for MT1.

Propositional Algebra and Proofs (Ch.1).

1. Propositions and connectives.
Connectives are functions of some propositional variables $f(P_1, \dots, P_n)$ taken proposintion depending of $P_1, \dots, P_n$ (their True values).

They define by their True tables.

2. Basis connectives (operations)

conjunction $\wedge$
disjunction $\vee$
negation $\sim$
Conditional $\Rightarrow$
biconditional $\Leftrightarrow$

Supplimentary operations
Scheffer $\square$
alternative disjuction

3. Propositional formula – composition (superposition) of ~~formulas~~ operations.

Don't forget on brackets which we need for reading of formulas.

4. 2 functions are equivalent

$$f(P_1, \ldots, P_n) \equiv g(P_1, \ldots, P_n)$$

(same variables!)

iff they have identical True tables.

$f \equiv g$ iff $f \Leftrightarrow g$ is Tautology.

2 ways to prove equivalency:

- by True tables
- using Lows of Propositional operations (Tautologies)

Remarks. Instead of complet tables we can use Truncated ones: ~~which can~~ compare only values of $P_1, \ldots, P_n$ on which f, g are True (or False).

5. We have several groups of Laws:

- basic laws for $(\vee, \wedge, \sim)$: commutaive, assotiative, distributive, de Morgen.
- absorption laws (like $P \vee (P \wedge Q) = P$, $P \vee (\sim P \wedge Q) = P \vee Q$.
- 2 groups for conditionals: direct and for contrapositionc.

Always give clear references whe make transformations.

6. 2 important types problems:
- find if $f(P_1, \ldots, P_n)$ is Tautology, ~~Con~~ Contradiction or Neither. At first 2 cases you need verify that f ~~is~~ takes the same values for all values of variables; on 3rd one – find 2 combinations on which is take T and F.
- transformation formulas to convenient form: when negations stay only for variables.

7. Complete systems of operations.

We can any connective $f(P_1, \dots, P_n)$ ~~repr~~ given by True table to represent by an equivalent formula with $\{\wedge, \vee, \sim\}$. So these 3 operations give a complet system of operations. Then we can transform at ~~a~~ formula with only $\vee, \sim$, either $\wedge, \sim$.

8. Correspondance between Propositional and Boolean functions.

9. Conditional, Biconditional, Theorems.

Properties of $a \Rightarrow B, a \Leftrightarrow B$.

Theorem $P \Rightarrow Q$; Inverse Theorem $Q \Rightarrow P$
$(P \Rightarrow Q \not\Leftrightarrow Q \Rightarrow P)$.

Proofs direct, by contradiction $((\sim Q \Rightarrow \sim P) \Leftrightarrow (P \Rightarrow Q))$

To prove $P \Leftrightarrow Q$ we need prove both: direct $P \Rightarrow Q$ and inverse Theorems $Q \Rightarrow P$.

10. Proofs of 3 Theorems: irrationality $\sqrt{p}$; infinity of prime numbers; numbers from only 5.

9b) $b = aq + r,\ 0 \le r < |a|$

$a = 5,\ b = 36$

$36 = 5q + r \Rightarrow q = 7,\ r = 1.$

~~Sect~~

b) $a = -8,\ b = -52$

$-52 = -8 \cdot q + r,\ 0 \le r \le 8$

$q = 7,\ r = 4$

Sect. 1.3.

1.. (c) $P(x)$ - x is isosceles

$Q(x)$ - x is ~~a~~ right

$\exists x \left(P(x) \wedge Q(x) \right)$

(d) $\forall x \left(Q(x) \Rightarrow \sim P(x) \right).$

(f) $P(x)$ x is honest

$\left(\exists x\, P(x) \right) \wedge \left(\exists y \sim P(y) \right)$

(g) $\forall x \quad x \ne 0 \Rightarrow (x > 0 \vee x < 0)$

(n) $P(x, y)$ x loves y

$\forall x\, \exists y\, P(x, y)$

(c) $\forall x \quad x>0 \Rightarrow \left(\exists y \quad x=2^{y} \wedge \forall z \left(x=2^{z} \Rightarrow y=z\right)\right)$

$(\forall x)\, x>0 \Rightarrow (\exists y!)\; x=2^{y}$.

5. $P(x,y)$ Person x dislikes taxes y

$\forall x \forall y\, P(x,y)$, $\forall x \exists y\, P(x,y)$, $\exists x \forall y\, P(x,y)$

$\exists x \exists y\, P(x,y)$.

6. b) T, c) T, d) T

8. a) g) F

9. a) For all natural x we have $x \geq 1$.

c) Each prime number $x \neq 2$ is odd.