Lecture 10.

1. Quiz 4.

2. HA 5.

Sect. 1.7.

2. a) 2 cases

n - even $\Rightarrow n^2, n, 5n^2, 3n, 5n^2+3n+4$ are even

n - odd $\Rightarrow 5n^2, 3n$ odd $\Rightarrow 5n^2+3n$ - even

(c) Possible remainders for 5 are $0,1,2,3,4$. Their sum is multiple 5.
Remainders of 5 consequtive number $\{0,1,2,3,4\}$ starting from one of them.
So their sum is multiple 5.

(e) $n^3-1n = n(n-1)(n+1)$ - product of 3 consequentive number.
Between them one multiple 3 and at least one is even $\Rightarrow 6 \mid n^3-n$

f) (not in HA)
$12 \mid (n^3-n)(n+2)$. Between $n, n-1, n+1, n+2$ there 2 even and one of them multiple 4.

4. (a) Contradiction

$(P \wedge \sim Q) \Rightarrow \sim R$ $\quad \sim Q$ - $x+y$ - rational

$(P \wedge \sim Q) \Rightarrow y = (x+y) - x$ - rational.

Contradiction.

(b) Take any rational z, ~~and~~ irrational x, and $y = z - x$. Then y is irrational, $z = x+y$. (More details!)

7a. $x > 0 \Rightarrow \frac{|2x-1|}{x+1} \leq 2$

$x > 0 \Rightarrow$ Equiv to: $|2x-1| \leq 2(x+1)$ (*)

2 cases: 1) $x \geq \frac{1}{2} \Rightarrow |2x-1| = 2x-1$

(*) $2x-1 \leq 2x+2 \Leftrightarrow -1 \leq 2$

2) $x < \frac{1}{2} \Rightarrow |2x-1| = -2x + 1$

(*) $\underset{\wedge 0}{-2x} - 1 \leq \underset{\vee 0}{2x} + 2$

For $x > 0$

9b) $b = aq + r, \quad 0 \le r < |a|$

$a = 5, b = 36$

$36 = 5q + r \Rightarrow q = 7, r = 1.$

~~Sect~~

b) $a = -8, \; b = -52$

$-52 = -8 \cdot q + r, \quad 0 \le r \le 8$

$q = 7, r = 4$

Sect. 1.3.

1. (c) $P(x)$ - x is isosceles

$Q(x)$ - x is ~~a~~ right

$\exists x \left(P(x) \wedge Q(x) \right)$

(d) $\forall x \left(Q(x) \Rightarrow \overline{P(x)} \right).$

(f) $P(x)$ x is honest

$\left(\exists x \, P(x) \right) \wedge \left(\exists y \, P(y) \right)$

(g) $\forall x \quad x \ne 0 \Rightarrow (x > 0 \vee x < 0)$

(n) $P(x,y)$ x loves y

$\forall x \, \exists y \, P(x,y)$

Extra problems.

1. a) $p=3$, $l=1$: $3|9$

$p=11$, $l=2$: $11|99$

$p=37$, $l=3$ $37|999$

b) $n = \overline{a_k \dots a_1 a_0}$

$= a_0 + a_1 10 + \dots + a_k 10^k$

$\forall m$ $10^m = 3q_1 + 1$

$10^m = 9q_2 + 1$

So for 3, 9 remainder of equal remainder of $a_0 + a_1 + \dots + a_n$.

c) Let $m = \overline{a_1 a_0} + \overline{a_3 a_2} + \dots + \dots \overline{a_{2p+1} a_{2p}}$

$n = \overline{a_1 a_0} + \overline{a_3 a_2} \cdot 100 + \overline{a_5 a_4} \cdot 100^2 + \dots$

Remainder m, n in the division on 11 coincide.

Proof. ~~$11 | 10^9$~~ $\forall s$ $11 | 100^s - 1$ ∎

2) Last digit must be even so it is 8.

First digit can't be 1 since $183 > 138$.

So # of last page is 318.

~~318 - 183 + 1~~ So # pages = 318 - 182 = 136