MATH 300. INTRODUCTION TO MATHEMATICAL REASONING.
FALL 2015.
WEEK 9 (LECTURE 16-17).
PREDICATS AND QUANTIFIERS. REVIEW FOR MT2.

1. Reading: Sections 1.3,1.6 and Lecture Notes.

2. Home assignment (Due Monday, November 2) (to submit). 1.Prove Tautologies

$$(\forall x)(P(x) \wedge Q(x)) \Leftrightarrow (\forall y)P(y) \wedge (\forall z)Q(z);$$

$$(\exists x)(P(x) \vee Q(x)) \Leftrightarrow (\exists y)P(y) \vee (\exists z)Q(z);$$

$$(\exists x)(P(x) \wedge Q(x)) \Rightarrow (\exists y)P(y) \wedge (\exists z)Q(z);$$

$$(\forall x)(P(x) \Rightarrow Q(x)) \Rightarrow ((\forall y)P(y) \Rightarrow (\forall z)Q(z));$$

but the following formulas are not tautologies (construct counterexamples):

$$(\exists x)(P(x) \wedge Q(x)) \Leftarrow (\exists y)P(y) \wedge (\exists z)Q(z);$$

$$(\forall x)(P(x) \Rightarrow Q(x)) \Leftarrow ((\forall y)P(y) \Rightarrow (\forall z)Q(z)).$$

2. Let us N is the product of 10 consequent natural number. Find maximal a, b, c such that all such N we have

$$2^a 3^b 5^c | N.$$

3. Using axioms prove that the set of points on any line is infinite; that there is infinite number points outside of any line.

4. Give a predicative definition of a parallelogram $xyuvv$. Prove that if the points x, y, u do not lie on one line then there is a parallelogram with these vertexes and such parallelogram is unique.

5. Prove that the product of 2 real numbers is negative iff exactly one of them negative.

6. Prove that $-x^2 - 2x + 15 > 0$ iff $-5 < x < 3$.